A Fuzzy Approach to Innovation in Tourism Clusters Using Galois Group Theory*

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Abstract

Tourism plays a decisive role in the development of cities and regions, however increasing competition between touristic destinations, globalization and new trends suppose novel challenges to decision and policy makers. A methodological structure towards grouping diverse highlights of the city of Morelia, Michoacán – México using Galois Group Theory and Fuzzy Logic is proposed. Results conclude that products with different characteristics, properties and peculiarities can be grouped over a structured and visual technique. The originality of the study relies on the capacity and flexibility of the model to group different highlights under subjective and uncertain conditions.

Keywords: Uncertainty, Fuzzy logic, Galois group theory, Fuzzy sets, Tourism clusters

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Innovación en Clusters Turísticos: Una Aproximación desde la Lógica Difusa y la Teoría de Grupos de Galois

Resumen

El turismo desempeña un papel decisivo en el desarrollo de ciudades y regiones, sin embargo, la competencia entre destinos turísticos, la globalización y tendencias cambiantes producen desafíos a tomadores de decisión y ceadores de políticas. Se propone una estructura metodológica para la agrupación de puntos de interés de la ciudad de Morelia, Michoacán – México utilizando la Teoría de Grupos de Galois y Lógica Difusa. Resultados concluyen que productos con diferentes características se pueden agrupar bajo una técnica estructurada y visual. La originalidad del estudio se basa en la flexibilidad del modelo para agrupar diferentes lugares bajo condiciones subjetivas e inciertas.

Palabras clave: incertidumbre, lógica difusa, teoría grupo de Galois, conjuntos difusos, clusters turísticos

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1. Introduction

Tourism plays a decisive role in economic development, it constitutes a driving and diversifying activity (see e.g. Mazanec *et al*., 2007; Gomezelj *et al*., 2008; Navickas *et al*., 2009; Crouch, 2010; among others). Nowadays tourism is generally considered a tool for economic development, by all its fundamental effects. Tourism has specifically been a fundamental factor in the economic development of Mexico, recording an upward trend accounting for 8.4% of national GDP and generating over 2 million direct jobs. Mexico occupies the 44th position in the world ranking of Tourism Competitiveness according to the World Economic Forum 2013. It stands in the 15th position of the worldwide ranking of international visitors, and around 24.151 million international visitors were reported to visit the county in 2013, capturing $13.8 billion dollars (Banxico, 2013).

However the positive figures that tourism activity has on Mexican economy such as globalization, variation on tourist's preferences, development of new products and the increasing long-distance voyages over the past ten years, have raised the competition among world touristic destinations Sectur (2007). In this highly uncertain environment, together with the degree of importance that touristic activities have on the development of the country, it remains essential that touristic destinations focus on being competitive, since “achieving optimal levels of competitiveness becomes an absolutely strategic factor”, (Solá, 1994).

Based on international experience, it has been detected that the development of industrial clusters and services (also known as business networks and local production systems), are important economic policy instruments to modernize and widely strengthen the competitiveness of a group of firms or industries. Porter (1990), mentions that a "cluster is a group of companies that operate through networks around a basic productive activity, concentrated geographically and/or sectorally by commonalities and complementarities around the basic sector of industry. They achieve a high degree of specialization, competitiveness and efficiency, generating dynamic processes through which it is possible to guarantee the success of the group”. The conformation of various business groups around the globe has given enough evidence on which some conditions must be met in order to encourage the concentration of companies and guarantee their operations under a scheme of competitiveness and continuous improvement. A key tool in shaping clusters is the "Diamond Model", (Porter, 1990). According to the author, there are four factors: strategy, structure and business rivalry, demand conditions, related and supporting industries.

Regional development could then benefit from the alignment of touristic clusters by the implementation of innovative strategies towards groups of highlights of a specific city explicitly designed for a visitor profile. However, the alignment and similarity between touristic destinations becomes essential for the correct development of a cluster. By failing in the alignment, the cooperation would not be reachable and the benefits could be scarce.

In order to identify the most affine members of a cluster we recourse to the definition provided by Gil – Aluja (1999) and "define the affinities as those homogeneous groups that at certain, ordered and structured level, link elements of two sets of different nature, related by the essence of the phenomena they represent". The main objective of this research is to apply a methodological structure towards grouping specific highlights on touristic destinations based on Galois Group Theory utilizing Fuzzy Logic tools for the treatment of information under uncertainty. The most linkable highlights could create groups, exerting the benefits of clusters. Such benefits contribute to the regional development of the city aiding decision and policy makers to the creation of robust innovative strategies based on systematic and efficient approaches.

In order to achieve the main objective, in section 2 we present the preliminaries i.e. the theoretical framework on which the paper relies. In section 3 we detail the application of Galois Group Theory and Galois Lattices in the specific environment of the studied city. Section 4 comprehends results obtained and discussions over remarkable topics. Lastly, section 5 presents the conclusions of the study.

2. Preliminaries

2.1 Touristic destinations

A touristic destination is defined as "a specific geographical area with distinctive climates, estate, infrastructure and services, and with some administrative capacity to develop common tools for planning. It acquires centrality by attracting tourists through structured products perfectly tailored to the desired satisfaction, all this by the enhancement and management of the attractions available, equipping it with a brand, which is marketed taking into account its integral character" Valls *et al.*, (2004). The author considers five features, a) an integrative motive, configured by its historical, geographical and anthropological characteristics; b) centrality, a territory that visitors aim to visit; c) perception, a structured offering of customer satisfaction services; d) brand, which represents the city in an appealing mode; e) must be provided with a joint marketing function.

Buhalis (2000) describes the main features that a touristic destination must have: attractions, accessibility, amenities, packs of available activities and auxiliary services. Depending on their organization and functioning, destinations can be classified into three types:

- One-to-one: in which there is no collaboration between tourism operators and the destination's management.
- The target model package: in this case, the activities and services the visitor receive are fully vested in the management of tourism operators, there is no user-enterpirse relation in the destination.
- The network target model: it is defined as a network of services, companies and proposals which are all linked together as a whole. Thus, a precise identity and a strong competitive capacity is generated.

2.2 Innovation towards tourism clusters

There is no manager or decision maker that could affirm that innovation does not carry competitiveness nowadays, it is, in some way, a given fact. (Porter, 1990) stated, "A nation's competitiveness depends on the capacity of its industry to innovate and upgrade. Companies gain advantage against the world's best competitors because of generating innovations".

Oslo Manual (2006) indicates that innovation can be characterized in four kinds: product innovation, process innovation, organizational innovation and marketing innovation: Product innovation implies significant changes of the characteristics in products or services. They can include completely new products or services and the significant improvement of existing ones. Process innovation refers to significant changes in production and distribution methods. Organizational innovations refer to the establishment of new methods of organization. These can be changes in the practices of the enterprises, in the organization of the workplace or in the external relations of the firm. Marketing innovation implies the establishment of new commercialization methods. These can include changes in the design and envelope of products, in the promotion and colocation of goods and in the methods of pricing in products or services.

In the present work we propose a methodology to aid decision makers in the marketing innovation process, by offering a specific grouping technique of products which has showed relevant results in practice. In recent years more attention has been attracted to this concept due to the fact that "the development of new marketing tools and methods plays an important role in the evolution of industries." (Chen, 2006).

Innovation, as for the endogenous and exogenous elements that surround them, presents high uncertain characteristics; therefore the adoption of decisions under a fuzzy approach has gained special relevance. Studies with a fuzzy-oriented standpoint have been increasing since the last century and have proven efficacy while dealing with complex phenomena.

2.3 Touristic clusters and competitiveness

The grouping of firms has allowed to obtain benefits such as access to new market segments and niches, technological improvement and know how, strengthening management capacity, increased ability to accelerate the learning process of human resources and organization, improving the quality of their products or services and reducing operating input costs, thus allowing them to gain competitive advantages.

Many authors have contributed to understand of clusters' characteristics; see e.g. Hill *et al*., 2000; Porter, 1998; Bell, 2005. Porter (1990) defines an industry cluster as "a set of industries related through buyer-supplier relationships or common technologies, common buyers, distribution channels or common workplace." The author describes two types of clusters: vertical and horizontal clusters. The firsts are made of industries linked through relationships between buyers and sellers, while horizontal groupings are the result of the existence of interconnections between companies of the same level, in order to share resources.

In recent definitions geographic concentration has been included as an important characteristic of a cluster. Authors like Jacobs & De Man (1996), propose three definitions related to the concept of a cluster. The regional cluster, composed of spatially concentrated industries, the sectorial cluster, composed of sectors or groups of sectors and the network clusters, integrated by value chains. The authors propose dimensions that can be used to define groups such as:

- The geographic clustering of the economic activity.
- Horizontal and vertical relationships between industries.
- The use of a common technology.
- Presence of a central actor; either a large company, research center, etc.
- The quality of interconnection between companies or cooperation between them.
- The presence of a central actor as a catalyst for a cluster.

Porter (2000) redefines the concept of cluster incorporating geographic concentration as an important element, as "geographic concentrations of interconnected companies, specialized suppliers, service providers, related industries firms and associated institutions; competing in particular fields but also cooperating". Such definition will be adopted for the purposes of the present research.

The OECD (2001) has defined the cluster as a grouping, or local productive system (LPS) in a same region that operate in similar business lines and whose relations foster the development of multiple interdependencies between them, strengthening their competitiveness in a wide variety of areas, such as: training, financial resources, technological development, product design, marketing, distribution and tourism.

It is necessary to consider some aspects such as their participation in the market, their industry experience, leadership and involvement in organizations and / or business associations for the election of the groups that should form the cluster. According to Sectur (2007), the correct evaluation of those aspects ensure the interaction and collaboration, thus, the evolution of the cluster to achieve expected benefits such as:

- Prioritizing collaboration over competition.
- Building and maintaining long-term inter-firm relationships.
- Attitude towards innovation and continuous improvement of tourism products and services.
- Self-generation of competitive advantages.
- Orientation to the achievement of high quality standards.
- High exchange of information among participating agents.
- Entrepreneurial capacity to cope with changes in regional, national and global economy.
- Flexibility to respond quickly to changes in the preferences and characteristics of tourists.

- Commitment of institutions to support the performance of clustering through infrastructure, legal framework, business climate, etc.
- Flexibility and efficiency of the supply chain by decentralizing operations.

In order to obtain the benefits previously stated, clusters must deal with several challenges: price battle, the balance of costs and benefits and inherent risks of cooperation, such as commercial secrets, technology, knowledge of markets and customers and opportunism. The state of collaboration is a complicated process since the benefits tend to be earned in a long and hypothetical term, while the risks and costs are immediate (Serret, 2011).

The benefits derived from the spirit of cooperation are the main attempt of a cluster, so it is very important to maintain a strategic focus and to be clear in the objectives pursed when entering into this kind of business techniques. It is important to clearly determine the vision in order to maintain a clear course on the evolution of the cluster (Serret, 2011).

Serret (2011), mentions that clusters contribute to economic growth; they facilitate an environment of innovation and entrepreneurship, which enables profitable growth through increased efficiency, quality and differentiation in goods and services. Such characteristics have a direct impact on competitiveness.

Multiple models on competitiveness of touristic destinations have appeared since the nineties. One of the main approaches comes from Ritchie & Crouch (2003), who define it as "the ability of a country to create added value and increase in this way, the national welfare by managing advantages and processes, attractions, aggressive and proximity, integrating the relationships between them in an economic and social model”.

A tourism cluster is determined by its ability to obtain superior benefits to the average of the industry at national and international level; the major impacts observed by the conformation of these groupings in terms of competitiveness according to the Mexican Ministry of Tourism (Sectur, 2007) are:

- Increased market appeal of destinations and products.
- Successful participation in the globalization process.
- Greater differentiation of tourism products at lower costs.
- Cost reduction over the whole industry chain.
- Scattering of knowledge and technology.
- Increased life cycle of tourism products through the incorporation or addition of attractions and services.
- Increased presence of clusters in the international arena.
- Ongoing strengthening of quality tourism products and services.
- Generating economies of scale with a high degree of flexibility.

Tourism competitiveness refers to an evolutionary behavior of companies, organizations and public institutions that interact within a region. Overall, ten elements can lead openly towards improving the competitiveness of a tourism cluster. These elements can be observed at an individual firm's level or at a global level cluster (Sectur, 2007):

- Predominance of the principle of collaboration on competition.
- Establishment of long-term inter-firm relationships.
- Innovation and continuous improvement of tourism products and services.
- Self-generation of competitive advantages.
- Orientation to the achievement of high quality standards.

- High exchange of information between participating agents, and even creation of Comprehensive Tourist Information Systems (ITIS).
- Entrepreneurial capacity to cope with changes in regional, national and global economy.
- Flexibility to respond quickly to changes in the preferences and desires of tourists.
- Commitment of institutions to support the performance of clustering through infrastructure, legal framework, business climate, etc.

2.4 The model

The model we build is founded on the principles of Galois Group Theory for the establishment of clusters and Fuzzy Logic for the evaluation and treatment of information under subjective and vague conditions. The model aims to modernize the methods previously used in the field of municipality's touristic management. It is a different and structurally improved way of establishing groups to create synergies. The optimal grouping can lead to join the most affine products in order to share capacity resources and, in general terms, help decision makers to create better strategies in order to increase the allurement, appeal and attraction of a city.

In our model, we elaborate a transition from verbal semantics to the corresponding numerical semantics using Fuzzy Logic tools in order to be able to group the most affine city's highlights, matching them by the valuation of their inherent characteristics, qualities and peculiarities. The model allows flexible procurement of information by empowering city experts and decision makers in the evaluation of a touristic place's characteristics and their desired similarity level.

The adequacy of the model is very important in terms of correctly measuring the characteristics of the city's highlights and determining if these characteristics can be matched with other highlights characteristics to build strong cluster-nature synergies. The characteristics are not always objective. The model we propose allows us to introduce subjective information for certain special cases where measurement is possible. Although there may exist some objective characteristics we have to accept the fact that the transition from verbal semantics to numerical semantics is subjective for those special cases that could have been measured, (Gil-Lafuente, 2002).

In general, the adoption and further application of Galois Group Theory as an extension of the Theory of Rings has multiple significances:

i) At first, it allows to establish different levels of synergies which could be created as of the inherent characteristics of the tourist attractions analyzed.

ii) Secondary, once the level of synergies has been established, the model allows to precisely know which are the specific characteristics that foster the optimization of the groups.

iii) Thirdly, the model allows the selection on which one of the characteristics the decision maker wants to prioritize in a specific environment and strategic requirements.

Galois Goup Theory has proven being efficient in different fields since the "order- or structure-preserving passage between two worlds of our imagination - and thus are inherent in human thinking wherever logical or mathematical reasoning about certain hierarchical structures is involved" (Denecke *et al*., 2004).

Other applications, which have conducted with success the application of Galois Lattices, can be found in the aggrupation of stakeholders for a better administration of enterprises (see Gil-Lafuente & Paula, 2013) and in human resources areas, with a personal selection model Keropyan & Gil-Lafuente (2013).

2.5 Fuzzy logic

It is widely accepted that decision-making process involves uncertainty, imprecision and imperfect or vague information. As stated by Bellman & Zadeh (1970) “much of the decision making in the real world takes place in an environment in which the goals, the constraints and the consequences of possible actions are not known precisely”. The theory of decision under uncertainty initializes with the publication of the first book from Kaufmann & Gil-Aluja (1986), and it has proven efficient considering over incomplete and uncertain knowledge information (see Ribeiro, 1996). The theory of Fuzzy Sets has been applied in the field of the formal sciences; nonetheless, in the past 44 years, researchers from all over the world have been publishing diverse research studies with applications in varied fields of knowledge.

The purpose of this work is to classify and group different products that could increase their attraction as a whole by the creation of synergies. The method to classify and group cities' highlights is based in Galois' Group Theory; see Keropyan & Gil-Lafuente (2013). In order to achieve so, we use tools derived from Fuzzy Logic, specifically the Theory of Fuzzy Sets to evaluate the inherent characteristics of variables and transit from verbal semantics to numerical semantics, (see Gil-Aluja *et al*., 2011). Such approximations admit us to build up a generalized model adapted to the conditions of expectancy and uncertainty.

3. Application

3.1 Studied city

In order to envision the application of the method, we will briefly describe the profile of the chosen city to develop the new touristic grouping model. As the reader will notice, some of the main reasons of choosing this specific metropolis settle down on the great importance that tourism has on its economy, the strategic location of the municipality and the need of fostering new management methods in order to maintain the attraction and appeal of the visitors. Morelia is a city located in the center of the Mexican Republic; the capital of the state of Michoacán de Ocampo. It is immersed in a huge region, known as "Greater Mexico City", where approximately 45 million people live and generate $290,000 million in LRP, more than a half of the whole nation, Florida (2008).

Geographically, Morelia finds itself 303 km from the capital of México City. Approximately 295 km towards the north we find the city of Guadalajara, Jalisco, well-known because of the culture, industry and the attractiveness to diverse businesses. About 196 km away from Morelia, we find the city of Querétaro, recognized because of all the industrial activities held out. One of the most important ports of the country, Lázaro Cárdenas port, finds itself around 387 km from the capital of the State. The city also connects to different metropolises of México thanks to its wide railroad infrastructure and the international Airport "General Francisco Mújica" (INAFED, 2010).

In economic aspects, the city displayed in 2008 an overall gross domestic product of $5,416.64 dollars, when the mean in the republic is $9,560 dollars (World Bank, 2013). The city raised its gross domestic product from 2003 to 2008 by 15% (INEGI, 2014). The main economic activities of the city are tourism, education and commerce. By 2010 the city reached 310,305 economically active citizens from whom 295,162 are employed (INEGI, 2014).

In terms of tourism, the city of Morelia is one of the first touristic destinations in the country due to its architectonical, cultural and historic legacy. The city also connects with a series of natural destinations, which increase the affluence of tourists. In 2009 the city displayed 116 lodging establishments and in the same year 723,318 tourists lodged in the city from which 97% were national tourists (INEGI, 2014). Referring to amenities, the city has a wide variety of theaters, museums, cinemas, bars and entertaining establishments, which nurture the popular culture and generate great attractiveness for the creation of micro and small enterprises (INAFED, 2010).

In terms of education, in 2011 Morelia offered 1,058 basic and medium level educational facilities, 3,895 classrooms, 81 libraries, 103 workshops and 165 laboratories. It is in this city where one of the most important universities of Mexico are established; the “Universidad Michoacana de San Nicolas de Hidalgo” where diverse ambits of science are studied such as health, administration and accountancy, legal, exact sciences, humanities, engineering and architecture. There is also a vast offer of postgraduate studies. Around 32,000 students are active in this university. An Institute of Technology resides in the city; around 4,650 students specialize in technological fields of the knowledge per year. In general terms there are 9 public institutes of advanced studies and 15 private ones (INEGI, 2014).

In terms of culture, the city of Morelia is a national exponent, having some of the most important artistic, musical and cinema events, as well as several expositions. Morelia has a total population of 729,279 inhabitants, with a 94.2% of alphabetization, and the standards of human development are among the top ranked of the country. We can find the “Conservatorio Musical” musical school in the city, founded in 1743, which provides the city of culture and artists in different disciplines. In other cultural aspects, Morelia was named Human Heritage in 1991 by the UNESCO, the association which also gave the city the name of “Sanctuary of the Monarch Butterfly”, and the sponsor of the “Day of the Dead National Celebration City”, “La Pirekua Musical Heritage of the Humanity”, and “Traditional Mexican cuisine - ancestral, ongoing community culture, the Michoacán paradigm” (INAFED, 2010; INEGI, 2014).

We have seen up to now the great cultural reservoir of the city, its great affluence of tourism and attractive popular amenities, but the development of the industrial tissue is incipient, and this shows up as an obstacle when providing rewarding employments to the citizens. Referring to the industry, the city has over 16 mining economical units, with approximately 100 active employees, as well as 3143 manufacturing enterprises with 14,606 active employees. In 2008 only 16 licenses for industrial land use were petitioned. The city has an industrial park where 180 enterprises offer around 9,000 employments. Most of the established enterprises just have distributing activity, and the manufacturing ones are small or medium sized companies. The whole data has been retrieved from the National Institute of Statistics and Geography (INEGI, 2014).

3.2 Establishing touristic highlights

In the application of the methodology, decision makers of the municipality of Morelia – México, need to group different highlights of the city for different tourist's profiles. By grouping the most interesting places of the city with common aspects, decision makers can choose from different strategies to maximize the experience a visitor may have.

In order to assess the specific challenge that the municipality of Morelia – México faces, we need to establish the products that will be offered to the visitor, on one hand, and the variables that will be held to evaluate the affinity in which the products relate to each other on the other hand. We now follow the steps specified in Gil Lafuente (2002).

The municipality of Morelia – México has a list of 13 highlights, which are considered as the prime touristic attractions of the city. We proceed to name them in the following table.

Table 1
Touristic highlights

A	Planetario
B	Palacio Clavijero
C	Catedral
D	Museo de Sitio Casa de Morelos
E	Jardín Villalongín
F	Monumento a José María Morelos
G	Callejón del Romance
H	Zoológico
I	Estadio Morelos
J	Teatro Melchor Ocampo
K	Centro Comercial Espacio las Américas
L	Museo Regional Michoacano
M	Bosque Cuauhtémoc

Source: elaborated from municipal touristic records (INAFED, 2010).

3.3 Formulating the variables

A list of characteristics of the places of interest was asked to be given. Each characteristic, singularity and peculiarity will be used as a basis for the creation of affinities for the whole group of products. The list was made by 10 assessors of the tourism office in the city. Their opinions were focused on the main characteristics that a city's visitor seeks. Once the head of the department approved the list, we proceed to name it.

(A) Historic: the level in which the highlight represents historical facts or events that happened in the city.

(B) Representative: the level in which the highlight remains on the memory of a visitor and serves for elucidating the city.

(C) Commercial: the level in which that specific highlight allows a visitor to generate economic spillover.

(D) Environmental: the level of green areas a specific highlight has, the greener the friendlier.

(E) Location: the distance a specific highlight has from the city's geographical center.

(F) Amenities: the level in which that specific highlight entertains the visitor, expositions, performances and cultural activities are some of the amenities included.

It is important to mention that the variables/characteristics included in the model are not exhaustive and have been treated with the same level of importance; we are currently working on further investigation, in which the nature of the variables and the importance of them affect, and apply certain weights in the model.

3.4 Grouping by affinities

The first step to conduct the process of grouping is to generate fuzzy subsets, valuating the different products due to their characteristics, singularities and peculiarities in the next way:

$$\underset{\sim}{i} = \begin{array}{|c|c|} \hline A & B \\ \mu_A^{(i)} & \mu_B^{(i)} \\ \hline \end{array} \cdots \begin{array}{|c|} \hline N \\ \mu_N^{(i)} \\ \hline \end{array}$$

$$i = a, b, c, \ldots, m$$

$$\mu_j^{(i)} \in [0,1], j = A, B, \ldots, N$$

Each product will be evaluated, according to its inherent characteristics, through a linguistic tag between 0 and 1 in which:

Table 2
Evaluation of variables

1	Excellent performance
0.9	Great performance
0.8	Really good performance
0.7	Good performance
0.6	Rather a better tan a poor performance
0.5	Nor a good or poor performance
0.4	Rather a poor tan a good performance
0.3	Poor performance
0.2	Really poor performance
0.1	Worst performance
0	Disastrous performance

Source: self elaborated.

In our case we have:

$\underset{\sim}{a}$ =

A	B	C	D	E	F
.3	.7	.4	.6	.4	.8

$\underset{\sim}{d}$ =

A	B	C	D	E	F
1	1	.2	.5	.8	.9

$\underset{\sim}{b}$ =

A	B	C	D	E	F
.7	.8	.2	.3	.9	.8

$\underset{\sim}{e}$ =

A	B	C	D	E	F
.5	.8	.3	.8	.7	.3

$\underset{\sim}{c}$ =

A	B	C	D	E	F
.8	1	.3	.7	1	.6

$\underset{\sim}{f}$ =

A	B	C	D	E	F
.8	.9	.1	.7	.7	.2

$$\underset{\sim}{g} = \begin{array}{cccccc} A & B & C & D & E & F \\ \hline \multicolumn{1}{|c|}{.6} & \multicolumn{1}{c|}{.6} & \multicolumn{1}{c|}{.6} & \multicolumn{1}{c|}{.8} & \multicolumn{1}{c|}{.7} & \multicolumn{1}{c|}{.7} \\ \hline \end{array} \qquad \underset{\sim}{k} = \begin{array}{cccccc} A & B & C & D & E & F \\ \hline \multicolumn{1}{|c|}{0} & \multicolumn{1}{c|}{.5} & \multicolumn{1}{c|}{1} & \multicolumn{1}{c|}{.2} & \multicolumn{1}{c|}{.4} & \multicolumn{1}{c|}{.7} \\ \hline \end{array}$$

$$\underset{\sim}{h} = \begin{array}{cccccc} A & B & C & D & E & F \\ \hline \multicolumn{1}{|c|}{.2} & \multicolumn{1}{c|}{.7} & \multicolumn{1}{c|}{.5} & \multicolumn{1}{c|}{1} & \multicolumn{1}{c|}{.5} & \multicolumn{1}{c|}{.9} \\ \hline \end{array} \qquad \underset{\sim}{l} = \begin{array}{cccccc} A & B & C & D & E & F \\ \hline \multicolumn{1}{|c|}{1} & \multicolumn{1}{c|}{.6} & \multicolumn{1}{c|}{.5} & \multicolumn{1}{c|}{.4} & \multicolumn{1}{c|}{.9} & \multicolumn{1}{c|}{.9} \\ \hline \end{array}$$

$$\underset{\sim}{i} = \begin{array}{cccccc} A & B & C & D & E & F \\ \hline \multicolumn{1}{|c|}{.1} & \multicolumn{1}{c|}{.8} & \multicolumn{1}{c|}{.4} & \multicolumn{1}{c|}{.5} & \multicolumn{1}{c|}{.3} & \multicolumn{1}{c|}{.8} \\ \hline \end{array} \qquad \underset{\sim}{m} = \begin{array}{cccccc} A & B & C & D & E & F \\ \hline \multicolumn{1}{|c|}{.5} & \multicolumn{1}{c|}{.5} & \multicolumn{1}{c|}{.4} & \multicolumn{1}{c|}{.9} & \multicolumn{1}{c|}{.8} & \multicolumn{1}{c|}{.7} \\ \hline \end{array}$$

$$\underset{\sim}{j} = \begin{array}{cccccc} A & B & C & D & E & F \\ \hline \multicolumn{1}{|c|}{.7} & \multicolumn{1}{c|}{.6} & \multicolumn{1}{c|}{.2} & \multicolumn{1}{c|}{.3} & \multicolumn{1}{c|}{.9} & \multicolumn{1}{c|}{.8} \\ \hline \end{array}$$

With this information we generate a fuzzy matrix comprehended by:

$$[\underset{\sim}{R}] = \begin{array}{c} \\ a \\ b \\ \\ m \end{array} \begin{array}{cccc} A & B & & N \\ \mu_A^{(a)} & \mu_B^{(a)} & \cdots & \mu_N^{(a)} \\ \mu_A^{(b)} & \mu_A^{(b)} & \cdots & \mu_N^{(b)} \\ \cdots & & \cdots & \cdots \\ \mu_A^{(m)} & \mu_A^{(m)} & \cdots & \mu_N^{(m)} \end{array}$$

In our case:

$\left[\underset{\sim}{R}\right] =$

	A	B	C	D	E	F
a	.3	.7	.4	.6	.4	.8
b	.7	.8	.2	.3	.9	.8
c	.8	1	.3	.7	1	.6
d	1	1	.2	.5	.8	.9
e	.5	.8	.3	.8	.7	.3
f	.8	.9	.1	.7	.7	.2
g	.6	.6	.6	.8	.7	.7
h	.2	.7	.5	1	.5	.9
i	.1	.8	.4	.5	.3	.8
j	.7	.6	.2	.3	.9	.8
k	0	.5	1	.2	.4	.7
l	1	.6	.5	.4	.9	.9
m	.5	.5	.4	.9	.8	.7

Once this information has been established and accepted, the decision maker must make a choice concerning the desired level of homogeneity the groups of highlights may have as for their specific characteristics, qualities and peculiarities.

Thus, for each characteristic we will establish:

$$0 \leq \theta_j \leq 1 \;\; , \mathrm{j} = A, B, \dots, N$$

In our case, the decision maker defined θ as:

$$\theta_A = 0.8, \theta_B = 0.8, \theta_C = 0.5, \theta_D = 0.3, \theta_E = 0.8, \theta_F = 0.6$$

Once the values of θ_j have been established, the valuations of each column of characteristics will be compared. If the valuation given to the specific characteristic is equal or superior to the desired level of homogeneity then the valuation is substituted with a 1, in the contrary 0. Specifically:

$$\mu_j^{(i)} \geq \theta_j \ , \ \beta_j^{(i)} = 1$$

$$\mu_j^{(i)} < \theta_j \ , \ \beta_j^{(i)} = 0 \ ,$$

$i = a, b, \dots, m$
$j = A, B, \dots, N$

By performing this action we will get a new matrix, in which the slots will only have 0 or 1. In our case

$[B] =$

	A	B	C	D	E	F
a				1		1
b		1		1	1	1
c	1	1		1	1	1
d	1	1		1	1	1
e		1		1		
f	1	1		1		
g			1	1		1
h			1	1		1
i		1		1		1
j				1	1	1
k			1			1
l	1		1	1	1	1
m				1	1	1

Source: self elaborated.

3.5 Maximum inverse correspondence algorithm

In order to find the most affine elements of the highlights of the city, we will follow the theory of affinities, specifically, the maximum inverse correspondence algorithm (Gil-Aluja, 1991). Studies and applications of this algorithm in economic and business sectors have led to excellent results while dealing with uncertain conditions.

1) From the set of highlights and characteristics, we choose the one that presents the fewer elements. In our case:

$$\{A,B,C,D,E,F\}$$

2) Create the "power set", which represents all the possible combinations of the set with the fewer elements. In our case:

{∅, A, B, C, D, E, F, AB, AC, AD, AE, AF, BC, BD, BE, BF, CD, CE, CF, DE, DF, EF, ABC, ABD, ABE, ABF, ACD, ACE, ACF, ADE, ADF, AEF, BCD, BCE, BCF, BDE, BDF, BEF, CDE, CDF, CEF, DEF, ABCD, ABCE, ABCF, ABDE, ABDF, ABEF, ACDE, ACDF, ACEF, ADEF, BCDE, BCDF, BCEF, BDEF, CDEF, ABCDE, ABCDF, ABCEF, ABDEF, ACDEF, BCDEF, ABCDEF}

3) Each element of the "power set" includes the corresponding elements of the set that hasn't been chosen for having a greater number of elements. The so called "connection to the right". In our case:

Ø	→	acbdefghijklm	ABC	→	Ø	ABCF	→	Ø
A	→	cdfl	ABD	→	cdf	ABDE	→	cd
B	→	bcdefi	ABE	→	cd	ABDF	→	Cd
C	→	ghkl	ABF	→	cd	ABEF	→	Cd
D	→	abcdefghijlm	ACD	→	l	ACDE	→	L
E	→	bcdjlm	ACE	→	l	ACDF	→	l
F	→	abcdghijklm	ACF	→	l	ACEF	→	l
AB	→	cdf	ADE	→	cdl	ADEF	→	cdl
AC	→	l	ADF	→	cdl	BCDE	→	Ø
AD	→	cdfl	AEF	→	cdl	BCDF	→	Ø
AE	→	cdl	BCD	→	Ø	BCEF	→	Ø
AF	→	cdl	BCE	→	Ø	BDEF	→	bcd
BC	→	Ø	BCF	→	Ø	CDEF	→	l
BD	→	bcdefi	BDE	→	bcd	ABCDE	→	Ø
BE	→	bcd	BDF	→	bcdi	ABCDF	→	Ø
BF	→	bcdi	BEF	→	bcd	ABCEF	→	Ø
CD	→	ghl	CDE	→	l	ABDEF	→	cd
CE	→	l	CDF	→	ghl	ACDEF	→	l
CF	→	ghkl	CEF	→	l	BCDEF	→	Ø
DE	→	bcdjlm	DEF	→	bcdjlm	ABCDEF	→	Ø
DF	→	abcdghijlm	ABCD	→	Ø			
EF	→	bcdjlm	ABCE	→	Ø			

4) We choose, from every non-void conjunct of the “connection to the right” the corresponding conjunct of the “power set”, which possesses the greater number of elements. In our case:

Ø	→	ABCDEF	bcdjlm	→	DEF
C d	→	ABDEF	cdfl	→	AD
L	→	ACDEF	bcdefi	→	BD
cdl	→	ADEF	ghkl	→	CF
bcd	→	BDEF	abcdghijlm	→	DF
cdf	→	ABD	abcdefghijlm	→	D
bcdi	→	BDF	abcdghijklm	→	F
ghl	→	CDF	abcdefghijklm	→	Ø

5) At this point, we have found the maximum number of relations, named affinities. The algorithm applied allowed to create the biggest amount of groups in an unambiguous method, due to the desired homogeneity level. In our case, the highlights of the city can be grouped in any of the specified conjuncts due to the characteristics, qualities and peculiarities they present.

The relations found between both sets create a Galois Lattice, which allows a demonstration in an ordered way of the homogeneous groups, as well as the perfect structuration of the elements.

3.6 Galois Group Theory and Galois lattices

Galois Theory is a connection between the field theory and the group theory. Certain problems in the field theory can be reduced into a group theory using Galois Theory. This allows an understanding upon the problems, making it easier to solve them. In the beginning, Galois used permutation groups to explain how the several roots of a given polynomial equation were related to each other (Edwards, 1984).

The Galois Theory is based on a remarkable correspondence between different subgroups of the Galois group of an extension *E/F* and intermediate fields between *E* and *F*.

If $G = \text{Gal}(E/F)$ is supposed to be the Galois group of the extension *E/F*. If *H* is a subgroup of *G*, the fixed field of *H* is the set of elements fixed by every auto-morphism in *H*, that is:

$$F(H) = \{x \in E : \sigma(x) = x \text{ for every } \sigma \in H\}$$

If K is an intermediate field, that is, $F \leq K \leq E$ define:

$$G(K) = Gal(E/K) = \{\sigma \in G : \sigma(x) = x \text{ for every } x \in K\}$$

In other words fixing group of K for *G (K)*, since *G (K)* is the group of auto-morphisms of E that leave K fixed. Galois Theory is about the relation between fixed fields and fixing groups, see Edwards, (1984); Artin, (1998).

3.7 Definitions of the theory

Following the definitions stated in Keropyan & Gil-Lafuente (2013):

Definition 1. A lattice is a partially ordered set in which two any elements have a least upper bound (LUB) and a greatest lower bound (GLB). A complete lattice is a lattice where any set has a LUB and a GLB.

Definition 2. A context K is a triple (O, F, ζ) where O is a set of objects; F is a set of attributes and $O \times F$ into $\{0,1\}$.

Definition 3. Given a context $K = (O,F, \zeta)$ let us define two mappings from *P(O)* into *P(F)* and from *P(F)* into *P(O)* using the same notation 0 by the formula:

$$\forall A \subset O, A' = \{f \in F \mid \forall o \in A, \zeta(o, f) = 1\}$$

$$\forall B \subset F, B' = \{o \in O \mid \forall o \in B, \zeta(o, f) = 1\}$$

A' Is called the dual of A, similarly B' is called the dual of B.

Definition 4. Given a context $K = (O,F,\zeta)$, the pair $C = (A,B)$ is called a concept of K if and only if $A' = B$ and $B' = A$.

Definition 5. A is called the extent of the concept C and B is called its intent.

This is denoted by $A = extend\ (C)$ and $B = intent\ (C)$.

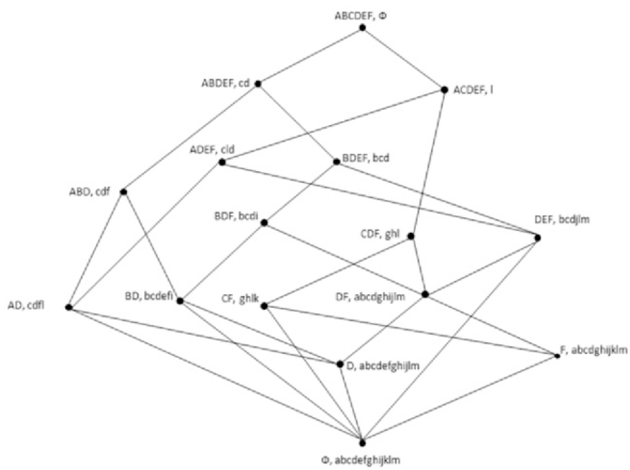
Considering an order relationship defined through inclusion of intents, one may define a Galois lattice or concept lattice.

Definition 6. The complete lattice *L (K)* of concepts of the context *K* is called (general) Galois lattice or concept lattice.

In our case we represent the Galois Lattice as follows:

Figure 1

Galois Lattice for the city's highlights case



Source: self elaborated.

4. Results and discussion

The algorithm finishes when we obtain the Galois Lattice, figure 1 represents in an ordered and systematic way, not only the total number of affinities that exist between the highlights of the city and the variables that comprise them, but it interconnects them in a coherent structure.

The decision maker can now opt for diverse combinations that could enhance the current schemes of touristic plans. In a first instance, we can see that it would be impossible to group all the tourist attractions because each highlight has different valuation on their specific characteristics, qualities and singularities. As the levels progress, we can visualize how the groups establish, in a first level highlights a, b, c, d, e, f, g, h, i, j, l and m all share the characteristic D or Environmental, in a similar way a, b, c, d, g, h, i, j, k, l and m all share characteristic F or Amenities. Perhaps the following levels could be more useful to the decision maker, since the amount of characteristics grow but the quantity of places of interest decrease. For example highlights c, d, f and i possess characteristics A and D, Historic and Environmental. The maximum number of characteristics possessed by a group of highlights is found in the top of the Galois Lattice representation, where c and d present singularities A, B, D, E and F.

The maximum number of groups have been presented in an ordered and structured way, thus, the decision maker has now the possibility to generate structured plans, following the levels of the Lattice. If the plan requires groups of highlights, which present A, D, E and F characteristics, then c, l, and d places are the most affine and could create better cluster-type synergies. Following the same idea, if the plans require D, E, and F characteristics then b, c, d, j, l and m are the most affine highlights to generate common strategies.

This result is highly interconnected with the level of homogeneity chosen by the experts; in this case, that level was the result of a specific profile of tourist. The decision maker could generate different scenarios, applying diverse combinations in order to get a full map of groups, depending on the various profiles of visitors that the city receives. Results obtained followed Galois Group Theory and the Galois Lattice, however further research needs to be conducted in order to extend the possibility to refine results; Co-Galois and the Galois Inverse Problem (see Záldivar, 1996) are acceptable techniques to achieving such task.

5. Conclusions

We propose an original group-based model methodology that relies on the comparison between determined variables collected by the inherent characteristics of different products, in order to create positive cluster-type interactions between them. The proposed model is originated from the basic principles of Galois Group Theory. This process allows grouping different products with a certain level of significance, detecting the level in which those groups could create synergies, and selecting which of the inherent characteristics of the products could be enhanced according to the specific needs and requirements of the decision maker.

The present work tries to shed a light on the academic world by offering a robust group based model in which subjective and relative factors are intrinsic for the decision making process. This analysis also tries to aid decision makers, so they can create common policies due to the results of the grouping processes.

Further research needs to be conducted, at a first instance, by studying the nature of the variables stated in order to know whether they need to be weighted. Conduct tests are then made to know if this weight plays a significant role on the results obtained, and applies the model in specific conditions. The model we present can be applied to different circumstances; we would like to encourage research on similar areas since it may allow optimizing the process of the products grouping under subjective and uncertain conditions.

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Disclosure policy

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