Rev. Cub. Física vol. 27 No. 1 (2010) p.89-93
ISSN: 0253-9268. Original paper

TECNOLASER 2009

Revista Cubana de Física
Calle I No. 302 e/ 15 y 17
Vedado, La Habana. CP 10400
www.fisica.uh.cu/biblioteca/revcubfi/index.htm

Polarization fluctuations in stationary light beams

A. Shevchenko[a], T. Setälä[a], M. Kaivola[a] y A.T. Friberg[a,b,c]

a) Department of Applied Physics, Aalto University, Espoo, Finland. andriy.shevchenko@tkk.fi†
b) Department of Physics and Mathematics, University of Eastern Finland, Joensuu, Finland.
c) Department of Microelectronics and Applied Physics, Royal Institute of Technology (KTH), Stockholm, Sweden. atf@kth.se
†corresponding author

Received15/04/2009. Approved in final version 4/29/2010.

Abstract. For fluctuating but statistically stationary beams of light the degree of polarization contains only limited information on time-dependent polarization. Two approaches towards assessing a beam's polarization dynamics, one based on Poincaré and the other on Jones vector formalism, are described leading to the notion of 'polarization time'. Specific examples of partially temporally coherent electromagnetic beams are discussed.

Sumario. En haces estacionarios, el grado de polarización sólo contiene información limitada de la dependencia temporal de la polarización. Para evaluar los cambios dinámicos de la polarización del haz dos métodos son descritos, uno basado en el formalismo vectorial de Poincaré y el otro en el de Jones, conduciéndonos al concepto de 'tiempo de polarización'. Ejemplos específicos de haces electromagnéticos coherentes parcialmente temporales son analizados.

Key words. Polarization 42.25.Ja, coherence 42.25.Kb, thermal radiation 44.40.+a.

1 Introduction

Beams of radiation generated by all sources (natural or artificial) exhibit polarization fluctuations, either due to the inherent features of the source or because of random variations in the medium. By characterizing the properties of these fluctuations, one can extract useful information about the source and the medium. The importance of polarization fluctuations has been addressed in several theoretical and experimental studies, such as on polarization mode dispersion in ordinary and specialty optical fibers,[1] super-continuum generation,[2] polarimetric radar imaging [3], vertical cavity surface emitting lasers,[4] atom-field interactions,[5] and polarimetry of cosmic waves, such as the microwave background radiation from early universe. [6]

However, the dynamical properties of polarization fluctuations, which distinguish the fields even when their degrees of polarization are the same, have not been directly studied. Here we use two different approaches to analyze the polarization-fluctuation dynamics of beam-like (2D) electromagnetic fields. This leads to the notion of 'polarization time'. We demonstrate the models by applying them to various particular cases, including partially polarized beams obeying Gaussian statistics and laser beams passed through an optical depolarizer.

2 Polarization fluctuations

The key point of our analysis is that the electric field vector of a statistically stationary and partially polarized electromagnetic beam fluctuates randomly in time, but at each instant of time the field is fully polarized. We wish to characterize the dynamics of these time-dependent polarization changes, while the usual 'degree of polarization' is constant. The degree of polarization only involves equal-time correlations and so it is independent of the beam's temporal coherence (and spectral) properties. We adopt two different approaches, one based on the Poincaré vector description and the other on the Jones vector formalism [7]. These two methods lead to physically similar results, so both can be employed. Indeed, they are also mathematically very closely connected, as we show.

Poincaré vector. The Poincaré vector $\mathbf{S} = (S_1,S_2,S_3)$ is a real-valued three-dimensional vector that consists of the Stokes parameters S_1, S_3, and S_3. When normalized by the optical intensity, i.e., the Stokes parameter S_0, we obtain the unit-length Poincaré vector $\mathbf{s} = \mathbf{S}/S_0$. For a stationary beam $\mathbf{s} = \mathbf{s}(t)$ varies randomly in direction, with anti-parallel directions representing orthogonal states of polarization, while its magnitude remains constant (= unity), regardless of the instantaneous values of the intensity of the beam. Thus the random function $\mathbf{s}(t)$ does not account for the intensity fluctuations.

However, the random function $\mathbf{S}(t) = \mathbf{s}(t)S_0(t)$ does – this quantity characterizes both the polarization and the intensity fluctuations, which may or may not be correlated. Thus, it is natural to employ the quantity $C(\tau) = \langle\mathbf{S}(t)\cdot\mathbf{S}(t+\tau)\rangle = \langle[\mathbf{s}(t)\cdot\mathbf{s}(t+\tau)]\, S_0(t)S_0(t+\tau)\rangle$ in the analysis of the fluctuation-induced changes of the polarization state and intensity of the beam. Clearly, the (fourth-order field) correlation function $C(\tau)$ has, in magnitude, the maximum value of $\langle S_0(t)S_0(t+\tau)\rangle$. Hence, the normalized correlation function for the characterization of the dynamics of the polarization fluctuations takes on the form [8]

$$\gamma_{\text{p, Poincaré}}(\tau) = \frac{\langle \mathbf{S}(t)\cdot\mathbf{S}(t+\tau)\rangle}{\langle S_0(t)S_0(t+\tau)\rangle} . \tag{1}$$

This parameter is a measure of the similarity of the polarization states at times t and $t+\tau$. It is obvious that $\gamma_{\text{p, Poincaré}}(0)=1$ and $-1\le\gamma_{\text{p, Poincaré}}(\tau)\le 1$. The upper and lower limits correspond to those cases in which the polarization states at time separations τ are the same and orthogonal, respectively. If the beam remains in the same polarization state, say a linear or circular polarization, then $\gamma_{\text{p, Poincaré}}(\tau)=1$ for all τ. When the polarization state changes in time, one may introduce a 'polarization time' τ_p [8,9], as illustrated in Fig. 1. The quantity τ_p is a characteristic measure of the time duration within which the electric field, on average, stays essentially in the same polarization state. Associated with τ_p is a 'polarization length' $l_\text{p} = c\tau_\text{p}$, where c is the speed of light, for the beam.

Jones vector. For a beam field the two-dimensional complex electric-field vector $\mathbf{E}(t)$ can be regarded as the Jones vector [2]. The second approach then is based on evaluating the mean value of the fraction, $\eta(t, t+\tau)$, of the beam's intensity left in the initial polarization state after time τ. A function, $\gamma_{\text{p, Jones}}(\tau)$, defined as the intensity-weighted, normalized average of $\eta(t, t+\tau)$, can be expressed in terms of the instantaneous Jones vectors $\mathbf{E}(t)$ and $\mathbf{E}(t+\tau)$ as [10]

$$\gamma_{\text{p, Jones}}(\tau) = \frac{\langle |\mathbf{E}^*(t)\cdot\mathbf{E}(t+\tau)|^2\rangle}{\langle I(t)I(t+\tau)\rangle} . \tag{2}$$

The function given in Eq. (2) has the general properties $\gamma_{\text{p, Jones}}(0)=1$ and $0\le\gamma_{\text{p, Jones}}(\tau)\le 1$. For a beam field that maintains its state of polarization we have $\gamma_{\text{p, Jones}}(\tau)=1$ for all τ, while for partially polarized beams $\gamma_{\text{p, Jones}}(\tau)<1$, when $\tau > 0$. Hence using Eq. (2) we may introduce a polarization time τ_p in full analogy with Fig. 1, except that in this case the quantity $\gamma_{\text{p, Jones}}(\tau)$ is allowed to fall off from its maximum value of unity, when $\tau = 0$, to a value of say 3/4 only. Physically both definitions are essentially the same. For this τ_p we naturally also obtain the corresponding polarization length, $l_\text{p} = c\tau_\text{p}$.[9]

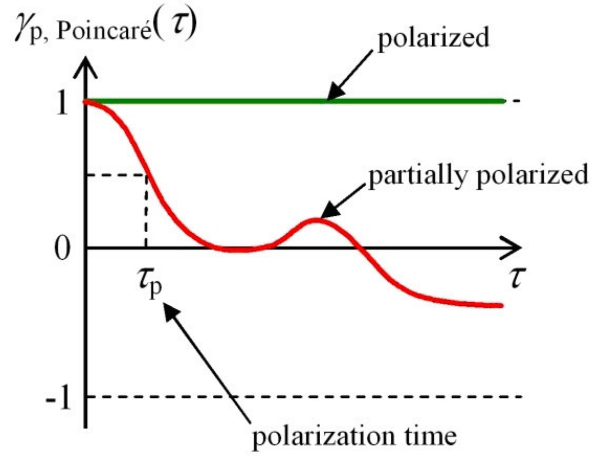

Figure 1. Illustrating the definition of polarization time for a partially polarized electromagnetic beam. The time separation during which $\gamma_{\text{p, Poincaré}}(\tau)$ falls to a relatively small value, say 1/2, defines τ_p.

3 Beams of gaussian statistics

For beams of Gaussian statistics the fourth-order field correlation functions in Eqs. (1) and (2) can be reduced to the second-order correlation functions by the use of the Gaussian moment theorem for complex-valued functions [7]. More explicitly, from Eqs. (1) and (2) we find [8,10]

$$\gamma_{\text{p, Poincaré}}(\tau) = \frac{P^2-\gamma^2_{\text{EM}}(\tau)+2|\gamma_\text{W}(\tau)|^2}{1+\gamma^2_{\text{EM}}(\tau)}, \tag{3}$$

$$\gamma_{\text{p, Jones}}(\tau) = \frac{1+P^2+2|\gamma_\text{W}(\tau)|^2}{2[1+\gamma^2_{\text{EM}}(\tau)]}, \tag{4}$$

where

$$P^2 = 2\frac{\text{tr}[\Phi^2(0)]}{\text{tr}^2[\Phi(0)]}-1 , \tag{5}$$

$$\gamma_\text{W}(\tau) = \frac{\text{tr}[\Phi(\tau)]}{\text{tr}[\Phi(0)]} , \tag{6}$$

$$\gamma_{\mathrm{EM}}^2(\tau) = \frac{\mathrm{tr}[\Phi(\tau)\Phi(-\tau)]}{\mathrm{tr}^2[\Phi(0)]} \, , \quad (7)$$

and (with $i,j = x,y$)

$$\Phi(\tau) = \left\{ \left\langle E_i^*(t) E_j(t+\tau) \right\rangle \right\} \quad (8)$$

is the 2x2 mutual coherence matrix [7]. Here, P is the beam's degree of polarization [7], $\gamma_W(\tau)$ is a complex electric correlation function (time-domain analogy to the intensity-fringe visibility) [11], and $\gamma_{EM}(\tau)$ is the electromagnetic degree of coherence [12].

It follows by simple algebra from Eqs. (3) and (4) that $\gamma_{\text{p, Poincaré}}(\tau) = 2\gamma_{\text{p, Jones}}(\tau) - 1$. Indeed, this result holds not only for stationary beams obeying Gaussian statistics, but one can by straightforward calculations directly prove that Eqs. (1) and (2), in general, satisfy this relationship [10]. This result thus shows that the two approaches are in full quantitative agreement. We observe that $\gamma_{\text{p, Poincaré}}(\tau) \to P^2$ as τ tends to infinity, while $\gamma_{\text{p, Jones}}(\tau) \to (P^2+1)/2$ in the same limit.

4 Examples

We demonstrate the usefulness of our formulations by several specific examples taken from the nature and from practical applications with laser radiation.

Gaussian correlated beams. Suppose that the mutual coherence matrix $\Phi(\tau)$ of the beam is of the form

$$\Phi(\tau) = \mathbf{J} \exp(-\tau^2 / 2\sigma^2) \, , \quad (9)$$

where $\mathbf{J}$ is the polarization matrix and σ characterizes the beam's coherence time. On substituting this into Eq. (3) we obtain [8]

$$\gamma_{\text{p, Poincaré}}(\tau) = \frac{2P^2 + (3+P^2)\exp(-\tau^2/\sigma^2)}{2 + (1+P^2)\exp(-\tau^2/\sigma^2)} \, . \quad (10)$$

This result is plotted in Fig. 2 for three values of P. It is seen that for substantially unpolarized beams (low values of P) the polarization time τ_p is approximately equal to the coherence time σ. For larger values of P the polarization time increases, while the coherence time of course remains unchanged.

Blackbody radiation pencils. Let us now assume that we draw a pencil of light form a blackbody radiation source at temperature T. The beam (in any direction) then is fully unpolarized [13] and it is described by the Planck spectrum [7]. From this information we may at once compute the mutual coherence matrix $\Phi(\tau)$ for the pencil. On substituting it into Eq. (4) we then find, after some algebra, that [10]

$$\gamma_{\text{p, Jones}}(\tau) = \frac{1 + 2\left| \frac{90}{\pi} \zeta\left(4, 1 + i\frac{k_B T}{\hbar}\tau\right) \right|^2}{2 + \left| \frac{90}{\pi} \zeta\left(4, 1 + i\frac{k_B T}{\hbar}\tau\right) \right|^2} \, , \quad (11)$$

where k_B is Boltzmann's constant, $\hbar$ is the Planck constant divided by 2π, and $\xi(s,a)$ is the generalized Riemann-Hurwitz zeta function [7].

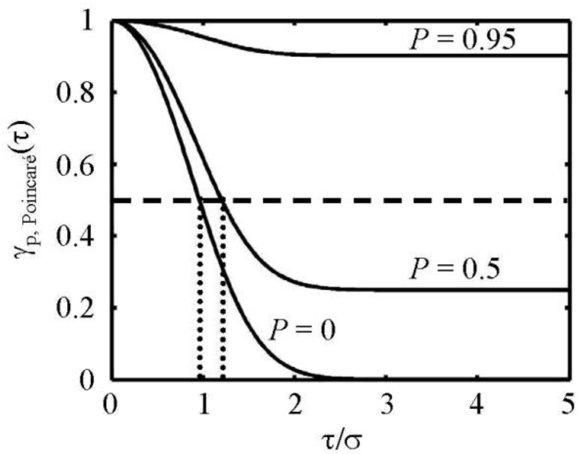

Figure 2. Behavior of $\gamma_{\text{p, Poincaré}}(\tau)$ for temporally Gaussian correlated and partially polarized beams, leading to definitions of the polarization time τ_p.

The function in Eq. (11) is illustrated in Fig. 3 for three temperatures T. We see that $\gamma_{\text{p, Jones}}(\tau)$ falls off faster for the higher temperatures. From the points at which $\gamma_{\text{p, Jones}}(\tau)$ intersects 0.75 (dashed line in the figure) we may read the corresponding polarization lengths l_p, i.e., about 4 μm, 39 μm, and 114 μm for temperatures 300 K, 30 K, and 10 K, respectively. As we have explained above, identical results would be obtained from the points where $\gamma_{\text{p, Poincaré}}(\tau)$ takes on the value 0.5.

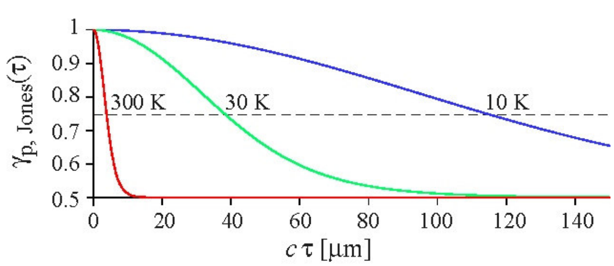

Figure 3. Illustration of $\gamma_{\text{p, Jones}}(\tau)$ for blackbody radiation beams at three different temperatures T. Intersections with the horizontal (dashed) line at 0.75 give the corresponding polarization lengths l_p.

If we consider that the sun is a blackbody source at temperature $T = 5800$ K, as is often done, we readily obtain an estimate of l_p = 200 nm for the polarization length of a beam of radiation from the sun. Likewise, the cosmic microwave background (CMB) radiation can, to a very high level of accuracy, be regarded as a blackbody field at temperature $T = 2.73$ K [14]. For a pencil of CMB radiation we then find a polarization length of l_p = 0.42 mm.[8,9]

Depolarized laser beams. As an illustrative practical example we consider an optical system that is used to depolarize a laser beam. Let us assume that a linearly polarized laser beam is split into two beams of equal powers and orthogonal (say, x and y) polarization states by making use of a polarizing beam splitter. The beams are allowed to propagate different distances, after which they are recombined into a single beam with another polarizing beam splitter. If the time delay, denoted by τ_d, of one beam with respect to the other is much longer than the coherence time σ of the incident field, the resulting beam can be considered unpolarized. If we further assume that the incident laser beam has a Gaussian time-domain correlation of the type in Eq. (9), i.e., that its degree of temporal coherence is given by $\exp(-\tau^2/2\sigma^2)$, we then find from Eq. (4) that [10]

$$\gamma_{\mathrm{p,\,Jones}}(\tau) = \frac{2 + 2e^{-\tau_d^2/\sigma^2} + 4e^{-\tau^2/\sigma^2}}{4 + 2e^{-\tau_d^2/\sigma^2} + e^{-(\tau-\tau_d)^2/\sigma^2} + e^{-(\tau+\tau_d)^2/\sigma^2}}. \quad (12)$$

In Fig. 4, this quantity is plotted as a function of τ/τ_d for several values of τ_d/σ, ranging from 0 to 20. As the time separation τ increases, all curves tend to the expression $[\exp(-\tau_d^2/\sigma^2)+1]/2$. Thus, the degree of polarization then is $P = \exp(-\tau_d^2/2\sigma^2)$, so that for sufficiently long delays τ_d the resulting beams indeed are unpolarized.

When $\tau_d/\sigma = 0$, the beam is fully polarized and the polarization time is infinite. For τ_d/σ slightly positive the function $\gamma_{\mathrm{p,\,Jones}}(\tau)$ still remains high enough so that the polarization time stays infinite. However, for larger values of τ_d/σ the polarization time is finite and it becomes shorter as τ_d/σ increases. Physically this means that the instantaneous polarization state evolves faster in time and it also deviates more from the average. Then, for very large values of τ_d/σ, the beam is essentially unpolarized and the polarization time τ_p is close to the coherence time σ.

We also observe that for large values of τ_d/σ the function $\gamma_{\mathrm{p,\,Jones}}(\tau)$ has a local minimum that is less than 1/2. This means that, on average, no matter what the beam's polarization state is at time t, at time $t + \tau$ the orthogonal polarization state will have a higher intensity. The time interval within which $\gamma_{\mathrm{p,\,Jones}}(\tau)$ is lower than 1/2 can be several coherence times σ.

When the delay τ_d increases, the local minimum of $\gamma_{\mathrm{p,\,Jones}}(\tau)$ moves towards $\tau/\tau_d = 1$. It follows from Eq. (12) that $\gamma_{\mathrm{p,\,Jones}}(\tau)$ then approaches the value of 2/5. Hence, on average, at time separation $\tau = \tau_d$, a fraction of 3/5 of the light intensity has shifted to a state of polarization that is orthogonal to the original (instantaneous) polarization state.

4 Conclusions

From our analysis it is evident that the two quantities, $\gamma_{\mathrm{p,\,Poincaré}}(\tau)$ and $\gamma_{\mathrm{p,\,Jones}}(\tau)$, can be used equally well to describe the dynamics of the fluctuating polarization state. In particular, they characterize how fast, on average, the instantaneous polarization state changes as a function of time. Furthermore, they yield different information on the polarization dynamics. Whereas the quantity $\gamma_{\mathrm{p,\,Poincaré}}(\tau)$ characterizes the average movement of the tip of the random temporal Poincaré vector on the Poincaré sphere, and connects the effective deviation of the tip from its average position to the degree of polarization, the quantity $\gamma_{\mathrm{p,\,Jones}}(\tau)$ instead directly describes the energy exchange between the field's (instantaneous) orthogonal polarization states. We have considered the properties of these two quantities and their implications in several specific examples that arise in nature or in practical applications. Both of these functions can be used to introduce and assess the notions of 'polarization time' and 'polarization length' for a beam field. These concepts are useful, characteristic measures over which the polarization state of a stationary (but random) beam field remains essentially unchanged. Examples such as manmade depolarized laser beams can readily be analyzed with the concepts discussed in this paper.

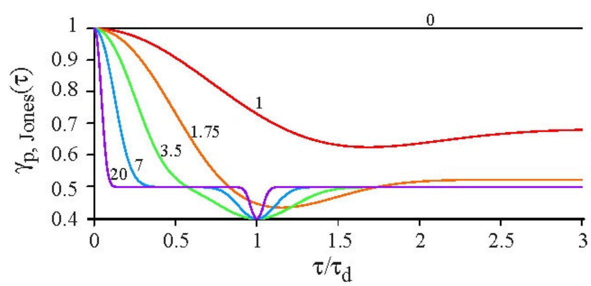

Figure 4. Behavior of $\gamma_{\mathrm{p,\,Jones}}(\tau)$ for a depolarized laser beam for several values of τ_d/σ, where τ_d is the delay of one of the beams in the superposition and σ is the coherence time of the original (fully polarized) laser beam. The parameters τ_d/σ range from 0 to 20.

Acknowledgments

This work was supported by the Academy of Finland (SA) and the Swedish Foundation for Strategic Research (SSF).

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