Relationship and comparison between partitioning coefficient and storativity ratio in a double porosity model

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Abstract

Although, well logs and well tests data investigate different volumes in the reservoir, their comparison can lead to a better understanding of the reservoir.

The storativity ratio and the partitioning factor estimate fluid storage ratio within the fractures in Naturally Fractured Reservoirs through different methods. The partitioning factor is estimated from well log data and the storativity ratio is estimated from well test analysis. By definition, these parameters are different, and only converge when the compressibilities of the fracture and matrix are equal, as presented in this paper.

A mathematical relationship is presented to obtain the storativity ratio from well log data, in order to be used as a critical parameter to design a pressure test, drawdown and/or build up test as an example, when there is not enough available information.

The fracture intensity index is related to these parameters and it is used to compare both parameters when the two factors are obtained independently. This is in order to obtain information not only from one source, but also to combine this information and be used to solve some problems present in naturally fractured reservoirs.

A field example is included to illustrate the comparison of the parameters and explain the utility of these arguments in a mature carbonate naturally fractured reservoir. The conclusion of this example demonstrates that there is a higher fracture intensity index away from wellbore, which is supported for an increasing in fracture intensity close to a fault.

Keywords: Naturally fractured reservoir, partitioning coefficient, double porosity model.

Relación y comparativo entre el coeficiente de partición y la relación de almacenamiento en un modelo de doble porosidad

Resumen

A pesar de que los registros geofísicos de pozo y las pruebas de presión investigan diferentes volúmenes en el yacimiento, la comparación de éstos puede llevar a un mejor entendimiento del yacimiento.

La relación de almacenamiento y el factor de partición estiman la relación de fluido almacenado dentro de las fracturas en los yacimientos naturalmente fracturados a través de diferentes métodos. Por definición, estos parámetros son diferentes y solo convergen cuando las compresibilidades de la fractura y matriz son iguales, tal y como se presenta en este artículo.
Se introduce una relación matemática para obtener la relación de almacenamiento por medio de la información de registros de pozos, con el fin de ser usado como un parámetro crítico para el diseño de pruebas de presión, decremento y/o incremento, por ejemplo, cuando no existe suficiente información disponible.

La intensidad de fracturamiento se relaciona con estos parámetros y se utiliza para compararlos cuando los dos factores son obtenidos de forma independiente. Esto permite obtener información, no solo por una fuente, si no también combinarla y utilizarla para resolver algunos de los problemas que se presentan en los yacimientos naturalmente fracturados.

Se incluye un ejemplo de campo para ilustrar la comparación de los parámetros y explicar la utilidad de estos factores en un yacimiento naturalmente fracturado maduro. La conclusión, en este caso, demuestra que existe una intensidad mayor en la lejanía del pozo, sustentada por un incremento en la intensidad de fracturamiento originado por una falla cercana.

**Palabras clave:** Yacimientos naturalmente fracturados, coeficiente de partición, modelo de doble porosidad.

**Introduction**

A fracture is a discontinuity that results from stresses that exceed the rupture strength of the rock (Stearns, 1990), therefore, a reservoir that contains fractures created by mother nature is a Naturally Fractured Reservoir, (NFR). The discontinuities can have a positive or negative effect on fluid flow.

Determination of the relative storage of matrix and fractures is one of most important aspects to characterize NFR. Nelson (2001), classified NFR in four different types according to its porosity and permeability.

- **Type 1:** Fractures provide the essential reservoir porosity and permeability.
- **Type 2:** Fractures provide the essential reservoir permeability.
- **Type 3:** Fractures assist permeability in an already producible reservoir.
- **Type 4:** Fractures provide no additional porosity or permeability but create significant reservoir anisotropy, (barriers).

The storage capacity coefficient or storativity ratio is defined as the fraction of total storage within the fractures divided by the total storage within the system, in a dual porosity system. This concept was introduced first by Warren and Root (1963), in conjunction with matrix/fracture permeability ratio, $l$. They conclude that these two parameters are enough to characterize the behavior of a double porosity system. The storativity ratio is given by:

$$
\omega = \frac{S_f}{S_f + S_m}
$$

...\(1\)

Where $S_f$ and $S_m$ are given by:

$$
S_f = \phi_f c_f h_f 
$$

...\(2\)

$$
S_m = \phi_m c_m h_m 
$$

...\(3\)
The partitioning factor, in a double porosity system, is defined as the ratio of secondary pore volume to the total pore volume. This concept was introduced by Pirson (1970), and is given by:

\[ \nu = \frac{\phi - \phi_b}{\phi(1 - \phi_b)} = \frac{\phi - \phi_m}{\phi} \quad \cdots(4) \]

These properties are linked to the fractured intensity index (FII), which is defined as the density of fractures per foot of formation. This attribute is a key factor for a quantitative prediction of the porosity and permeability of a NFR. Furthermore, it is directly related to the reservoir productivity and can be used to optimize reservoir management decisions. The fractured intensity index can be defined as:

\[ FII = v\phi \quad \cdots(5) \]

To characterize these types of reservoirs we have to take into account all the available information from well logs and transient pressure analysis. The partitioning factor can be determined using the total porosity and total resistivity from well logs. The calculation of the storativity ratio is carried out using well test analysis for a double porosity model. An explanation of the calculation for these two parameters is presented in the theoretical section.

**Theory**

The storativity ratio can be estimated from pressure transient analysis including type curve matching. For a drawdown test:

\[ \sigma = 10^{-Dp/m_1} \quad \cdots(6) \]

From equation 5, the value of Dp is the vertical separation between the parallel straight lines and m1 is the value of the slope as shown in Figure 1. This parameter can be also estimated by:

\[ \sigma = \frac{t_1}{t_2} \quad \cdots(7) \]

Values of \( t_1 \) are given at any time on the first straight line and \( t_2 \) is the time of the second straight line at the same pressure, as shown in Figure 1. The storativity ratio can be calculated also using the hydraulic diffusivity of the composite system and the hydraulic diffusivity of the fracture as follows:

\[ \sigma = \frac{\eta_c}{\eta_f} \quad \cdots(8) \]
The partitioning coefficient is a fundamental parameter used in the petrophysics characterization of NFR from well logs data. Archie (1949), introduced the following equations to evaluate the formation from resistivity and porosity well log data:

\[ S_w = I(\frac{-1}{a}), \quad \text{(9)} \]

\[ I = \frac{R_i}{FR_w} = \frac{R_i}{R_o}, \quad \text{(10)} \]

\[ F = a\phi^m = \frac{R_o}{R_w}, \quad \text{(11)} \]

Equations 8, 9, and 10 can be rearranged to obtain the following expression:

\[ \log(R_i) = -m \log(\phi) + \log(aR_w) + \log(I) \quad \text{(12)} \]
A log-log cross plot of porosity versus resistivity, pickett plot, and equation 7 can be used to evaluate a formation from well log data, as shown on the schematic representation in Figure 2. For the intervals with water saturation equal to 100%, a straight line can be drawn.

The value of the cementation exponent $m$ used in water saturation calculations, can be obtained from the straight lines shown in this figure. When the value of water saturation and porosity are equal to 100%, a $R_w$ can be calculated as shown in Figure 2.

Aguilera and Aguilera (2003) developed a dual-porosity model from Archie’s equation, considering a system with two different porosities, matrix and fractures or matrix and vugs, as follows:

$$
\phi^{-m} = \frac{1}{\left[ v\phi + (1 - \phi)/\phi^{-mb} \right]}
$$

...(13)

\[\text{Figure 2. Schematic representation of pickett plot.}\]

The partitioning coefficient, defined by equation 4, can be calculated from equation 12, using the value of $m$ obtained from pickett plots, the porosity of each interval and the matrix–block porosity.

**Mathematical relationship**

The partitioning coefficient can be determined in terms of the storativity ratio. Substituting equation 2 and 3 in 1 we obtain:

$$
\sigma = \frac{\phi_f c_f h_f}{\phi_f c_f h_f + \phi_m c_m h_m}.
$$

...(14)
For convenience, dividing both sides by the distance between fractures:

\[ \sigma = \frac{\phi_f c_f h_f}{h_m + \phi_m c_m h_m} \]  
\[(15)\]

We also know that the relationship between the total fracture width divided by the distance between fractures is equal to the fracture-block porosity attached to the bulk volume of the composite system:

\[ \phi_2 = \frac{h_f}{h_m} \]  
\[(16)\]

The porosity of the fracture attached to a single point properties is equal to 1, therefore, equation 14 can be written as:

\[ \sigma = \frac{\phi_2 c_f}{\phi_2 c_f + \phi_m c_m} \]  
\[(17)\]

Solving for the porosity of the fracture-block attached to the bulk

\[ \phi_2 = \frac{\sigma \phi_m c_m}{(1 - \sigma) c_f} \]  
\[(18)\]

Substituting equation 17 in equation 4

\[ \nu = \frac{\sigma \phi_m c_m}{(1 - \sigma) \phi c_f} \]  
\[(19)\]

Solving for the storativity ratio

\[ \sigma = \frac{\nu \phi c_f}{\phi_m c_m + \nu \phi c_f} \]  
\[(20)\]

Equation 19 presents a relationship that can be used to design a drawdown or buildup pressure test when there is not enough information. This situation can be presented in an exploratory well, before the pressure test is run, the only information is provided by the perforation, and well logs. This could lead to an optimization of the equipment time necessary for testing. The recommended time for a pressure test is ten times \( t \).
If the fracture and matrix compressibilities are the same, as commonly assumed, equation 16 demonstrate for this case that partitioning coefficient and storativity ratio are equal.

\[
\sigma = \frac{\phi_2}{\phi_2 + \phi_m} = \nu. \quad \text{(21)}
\]

The fracture intensity index, can be also written as a function of the storativity ratio, substituting equation 19 in equation 5 we obtain:

\[
FII = \frac{\nu \phi_m c_m}{(1-\nu) c_f} \quad \text{(22)}
\]

**Comparison fracture storativity ratio versus partitioning coefficient**

If these two parameters are independently calculated, partitioning coefficient from well log analysis and fracture storativity ratio from well test analysis, a comparison of these attributes can lead to obtain information of the areal extent of the secondary porosity distribution.

Engler (1996), compared both parameters assuming a direct relationship between them. This comparison can not be done because, by definition, these parameters are not equal. In this paper, the introduction of the fracture intensity index calculated from both parameters is compared to identify the variation of secondary porosity through the reservoir.

Fracture intensity index can be calculated from equation 5 for partitioning coefficient \((FII_{\phi})\) and from equation 21 for storativity ratio case \((FII_{\nu})\). These values can be compared to identify the variation of the second porosity through the area of study.

If the values of \(FII\) for both parameters are equal, \(FII_{\phi} = FII_{\nu}\), the values of the fracture intensity will be constant through the well log and well test volume. This will indicate that the reservoir is not highly heterogenous and the values used for matrix and fracture storage applies for the radius of investigation. A schematic representation of this case is provided in Figure 3.

**Figure 3.** Schematic representation for case.
The second case considers the value of $FFI$ calculated for the storativity ratio lower than the value of $FII$ partition factor, $FII_v > FII_w$, this considers that the fracture intensity close to the wellbore is bigger than the fracture intensity away from the wellbore, this case is highly unlikely. From a measurement viewpoint, the a secondary porosity response on well test would be rapid and probably masked by wellbore storage, thus increasing the difficulty of ascertaining results. It is recommendable to review the values of storativity ratio collected from the well test. A schematic representation is provided in Figure 4.

![Figure 4. Schematic representation for $FII_v > FII_w$ case.](image)

The last example considers the value of $FII$ calculated for the storativity ratio bigger than the value of $FII$ partition factor, $FII_v > FII_v$. This case suggests an increase in the secondary porosity getting away from the wellbore. The example provided in Figure 5 presents an increasing secondary porosity close to a fault with a decreasing in double porosity as getting away from the fault. Another possibility for this case could be an increasing in dissolution, thus increasing in vugs and solution channels along an apex of a structure in a carbonate formation where percolation of groundwater had occurred in the past.

![Figure 5. Schematic representation for $FII_v > FII_v$ case.](image)
Field example

The field example is a mature carbonate and naturally fractured reservoir in the upper Jurassic and Cretaceous periods, located in Eastern Mexico, general reservoir characteristics are presented in Table 1. The reservoir has an extension over 200 km$^2$, but the test and validation are only made in one block, presented in Figure 6.

### Table 1. Reservoir properties.

<table>
<thead>
<tr>
<th>Rock type</th>
<th>Limestone and dolomite</th>
<th>Oil gravity</th>
<th>28-31 API</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity</td>
<td>4-6 %</td>
<td>Pressure</td>
<td>1,200 psia</td>
</tr>
<tr>
<td>Fluid type</td>
<td>Black oil</td>
<td>Temperature</td>
<td>260 °F</td>
</tr>
</tbody>
</table>

![Figure 6. Structural map of the reservoir block, top of the Jurassic Kimmeridgian.](image)

The well proposed for this case was perforated in 2000 and completed in the Jurassic Kimmeridgian age; the well logs from completion are presented in Figure 7. The formation has low quantity of shale and the net gross is 180 meters. The calculations to obtain the partitioning factor are presented in Appendix A. The average partitioning factor for this well was 0.29 and average porosity of 0.047, from equation 5 we can calculate the $FII$ as follows.

$$FII = v\phi = 0.29 \times 0.0047 = 0.014$$
In 2006, an injection well test was carried out in the well to evaluate as a possible nitrogen injector for an immiscible project. The pressure transient data is presented in Figure 8. The match for this falloff test was done with classic wellbore storage (CWS), radial composite flow, and infinite acting limit. Table 2 presents the main parameters through this model. Figure 9 presents the final match with the main parameters where was calculated a storativity ratio of 0.4. File data is provided for more information.
Assuming equal fracture and matrix compressibilities, matrix porosity equal to 0.04, and storativity ratio of 0.4, we can calculate $FII$ from equation 21:

$$FII = \frac{\omega \phi_m C_m}{(1-\omega)C_f} = \frac{0.4 \times 0.04}{(1-0.4)} = 0.026$$

As we can see the calculated $FII$ from storativity ratio is bigger than the $FII$ from partitioning factor. This indicates the increase of the fracture intensity away from the wellbore. An important fault is close to the well as it is observed in the structural map in Figure 6. However, the values of the fracture intensity index do not have a big difference between them, and we can conclude the reservoir around the wellbore. An improvement in the injection index can be obtained from an acid stimulation to connect the highly fractured zone to the well.
Conclusions and recommendations

Even though, the partitioning factor and the storativity ratio are not the same by definition, a mathematical relationship can be derived to obtain the storativity ratio from the partitioning factor when there is not enough information available. The inclusion of this information in pressure test design is critical to conduct an optimum well test, which would reflect the characteristics of natural fractured reservoirs.

The comparison of well log data and pressure transient analysis can lead to a better understanding of the reservoir and well behavior. It can also be used to enhance the hydrocarbon recovery through hydraulic fracturing, acid stimulation or to place a new infill well based on the provided information.

In future work, more well data has to be included to validate the comparison of these two parameters and sustain the information presented in this work.

Nomenclature

\[ h_m = \text{Distance between fractures} \]
\[ I = \text{Resistivity index} \]
\[ \lambda = \text{Matrix/fracture permeability ratio} \]
\[ m = \text{Porosity exponent} \]
\[ m_I = \text{Semi log straight line slope} \]
\[ m_k = \text{Porosity exponent of the matrix block} \]
\[ n = \text{Water saturation exponent for the composite system} \]
\[ \eta_f = \text{Hydraulic diffusivity} \]
\[ R_o^* = \text{Resistivity of the matrix system at reservoir temperature when is 100\% saturated with water of resistivity} \]
\[ R_t = \text{True resistivity} \]
\[ R_w = \text{Resistivity of the water at reservoir temperature} \]
\[ S_m = \text{Storativity of the matrix} \]
\[ S_f = \text{Storativity of the fracture} \]
\[ S_w = \text{Water saturation} \]
\[ t = \text{Time} \]
\[ \nu = \text{Partitioning coefficient} \]
\[ \omega = \text{Storativity ratio} \]

Acknowledgments

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Appendix A

Calculation to obtain the partitioning factor of the field well example.

Resistivity and Porosity were obtained from the well log data, and averages of porosity and permeability were estimated each five meters intervals and plotted into a Pickett plot to obtain the porosity exponent of the system. From previous studies, an average of porosity exponent of the matrix is \( m_k = 2.0 \). Figure 10 presents the picket plot for this case.
The calculated $m$ for this case is equal to 1.362, using the values from well logs and equation 12, the partitioning factor for each interval was calculated. This was through an iterative method, Figure 11 presents the partitioning factors calculated as a function of depth.

The average of the partitioning factor is taken as representative of the well and was used for the calculations of the field example. All data are provided in the attached Excel file.
References


Semblanza del autor

**Antonio Jonathan Vázquez Zamora**

Es Ingeniero de yacimientos en el Complejo Antonio J. Bermúdez de Pemex Exploración y Producción, en el proyecto de mantenimiento de la presión de inyección de nitrógeno inmiscible y las estrategias de depósito en un campo de carbonato maduro. Su interés de investigación incluye el proceso EOR, los yacimientos naturalmente fracturados y la simulación de yacimientos. Tiene una Licenciatura de la UNAM y una Maestría de la Universidad de Calgary, ambos en Ingeniería Petrolera.