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TUNING EQUATIONS SET FOR FUZZY PI CONTROLLERS BASED ON FOPDT MODEL IDENTIFICATION AND MINIMUM IAE AND OUTPUT VARIANCE MINIMIZATION

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Abstract: A new set of tuning equations for PI-Fuzzy Logic Controllers based on FOPDT models were developed. A multilevel factorial experiment was designed and its output variable was a cost function based on IAE (Integral Absolute Error) and controlled variable variance. The proposed tuning equations set were obtained by developing regression models to optimum tuning parameters for every experimental condition. Regression models and variance analysis were statistically tested for significance. Dynamic simulations tests were carried out in order to compare the performance of two PI-FLC architectures based on tuning equations developed in this research and those available in the literature. Results demonstrate an improved performance by using a 2-membership functions FLC tuned with this new set.

Keywords: *PI-Fuzzy Logic Controller; Tuning Equations; Optimal Gain; FOPDT model.*

Resumen: Se ha desarrollado un nuevo conjunto de ecuaciones de sintonía para controladores PI difusos basado en la respuesta dinámica de proceso como un modelo de primer orden más tiempo muerto (FOPDT). Se diseñó un experimento factorial cuya variable de salida corresponde a una función de costo basada en la integral del valor absoluto del error (IAE) y la varianza de la variable controlada. El conjunto de ecuaciones de sintonía propuesto se obtuvo mediante el desarrollo de modelos de regresión que mejor ajusten los parámetros de sintonía

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óptimos para cada condición experimental. Los modelos de regresión obtenidos y el análisis de varianza fueron verificados estadísticamente para determinar significancia. Se llevaron a cabo simulaciones dinámicas con el fin de comparar el desempeño de dos arquitecturas de controladores **PI** difusos sintonizados con las ecuaciones desarrolladas en la presente investigación y otras disponibles en la literatura. Los resultados demuestran una mejora en el desempeño al sintonizar con el conjunto de ecuaciones propuesto, así como con la utilización de controladores basados en lógica difusa (FLC) con dos funciones de pertenencia en su arquitectura.

Palabras Clave: Controlador **PI** basado en lógica difusa, Ecuaciones de sintonía, Ganancia optima, modelo de primer orden más tiempo muerto (FOPDT).

1. INTRODUCTION

During the past several years fuzzy control has become one of the most active and fruitful fields for research in the application of fuzzy set theory (Woo, Chung, & Jin-Jye, 1998). The fuzzy control strategy considered in this paper is based on feedback control. Fuzzy logic controllers (FLCs) are rule-based systems which are useful in the context of complex processes, especially those who can be controlled by a skilled human operator without knowledge of their underlying dynamics (Herrera, Lozano, & Verdegay, 1995). The basic structure of FLC proposed by Mandami (Fig. 1) consists of four conceptual components, namely: the knowledge base, the fuzzification module, the inference engine and the defuzzification module [3,4]. The knowledge base system contains all the knowledge including a fuzzy control rule base and a database comprising facts, terms and concepts (Lucena, Palma, Cardozo, & Gil, 2012). The inference engine performs inference procedures upon the fuzzy control rules and given conditions to derive a reasonable control action (Lucena, Palma, Cardozo, & Gil, 2012). The fuzzyfication module defines a mapping from a real-value space to a fuzzy space, while the defuzzification module implements a mapping from a fuzzy space defined over an output universe ofdiscourse to a real-value space (Feng, 2006).

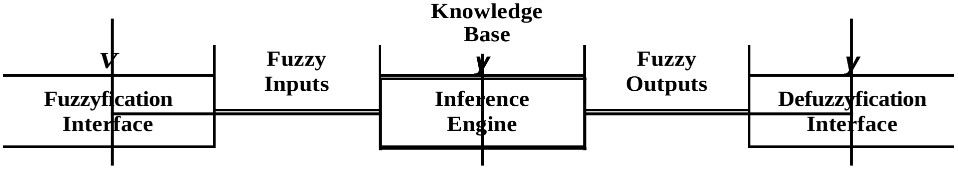


Figure 1. General structure of a Mandami-type Fuzzy Logic System

Although the application of fuzzy logic in designing a controller has many advantages compared with other methods (van der Wal, 1995), PID controllers are still the most widely used in industrial control loops worldwide. This is because PID controllers have simple structures, can be designed easily, offer good control system performance at acceptable cost(Precup & Hellendoorn, 2011), and there is a wealth of tuning methods available. However, FLCs are more robust than conventional PID controllers and their performance is less sensitive to parametric variations of systems orto unmeasured disturbances (Lucena, Palma, Cardoso, & Gil, 2012) because fuzzy controllers are composed by rules of conditional linguistic statements on the relationship between the input and output variables, and this provides them with the advantage of emulating the behavior of a human operator while dealing with model uncertainty(Woo, Chung, & Jin-Jye, 1998).

In the literature there are many categories of Fuzzy Logic Controllers based on the differences of fuzzy control rules and their generation methods, but this article it will focus on PI-type FLCs in order to develop a new set of tuning equations based on FOPDT models.

2. PI — Fuzzy Controller Structure

Let us consider a two-dimension PI fuzzy logic controller structure, in which error *e(t)* and change in the error *Ae(t)* are selected as input variables, while de output from the fuzzy logic system is chosen as the increment of control action represented by the signal to the control valve m(t) [3,7,8] . In Figure 2 it is shown the PI-FLC block diagram, used in this paper.

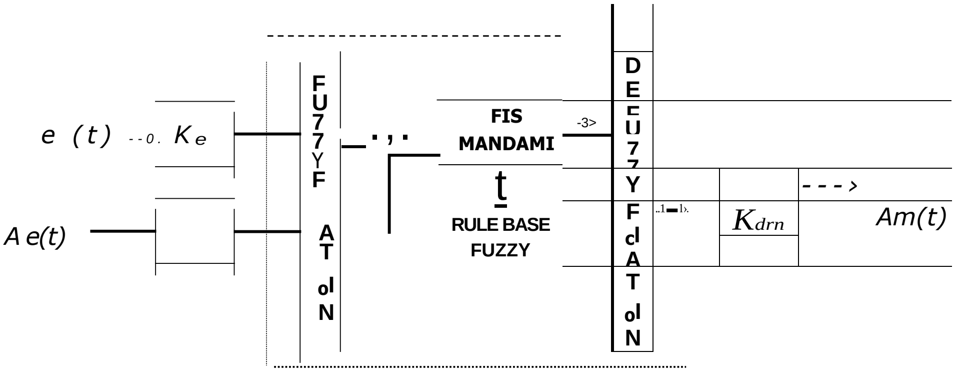


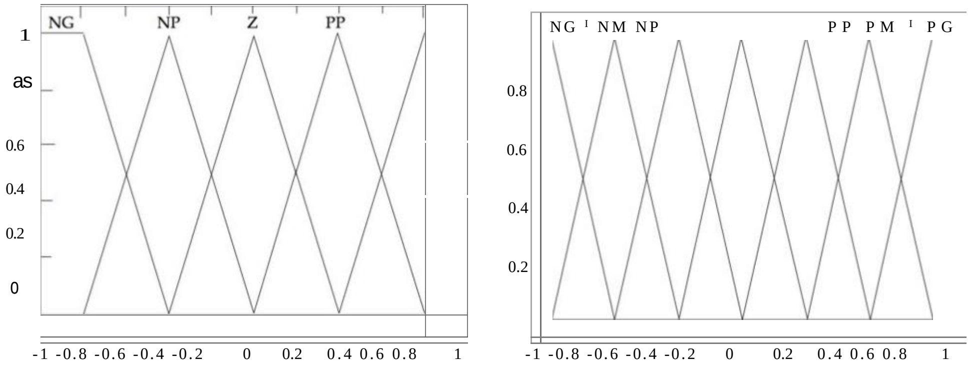
Figure 2. PI — *Fuzzy Logic Control schematics*

For the design of the PI-Fuzzy Controller, the linguistics variables are: e(n), *A* e(n) and *Am (n)* where n is the current discret time. The norma -lized universe of discourse for é and *A* e were chosen to be [-1.0,1.0] and partioned into five fuzzy sets, namely: Negative Big (NG), Negative Small (NP), Zero (Z), Positive Small (PP) and Positive Big (PG). For the fuzzy controller output, *Am,* the corresponding universe of discourse was defined in the range [-1.0,1.0] and partioned into seven fuzzy sets, namely: Negative Big (NG), Negative Medium (NM), Negative Small (NP), Zero (Z), Positive Small (PP), Positive Medium (PM) and Positive Big (PG). The membership function of e, de and Am are presented in Figure 3.

The rule base used in this work comprises twenty-five rules (see Table 1) based on an invert action controller(Gutiérrez & Sanjuan, 2006). For direct action, the sign of the rules scaling parameter must be change.

Table 1. *Format of the Rule Base used (Gutiérrez & Sanjuan, 2006)*

❑m(n)	*❑é(n)*				
	NG	NP	Z	PP	PG
NG	NG	NG	NM	NP	Z
NP	NG	NM	NP	Z	PP
é (n) Z49	NM	NP	Z	PP	PM
P	NP	Z	PP	PM	PG
P P	Z	PP	PM	PG	PG



(a) **(b)**

Figure 3. *Membership Functions (Gutiérrez & Sanjuan, 2006) for (a) Input Variables (b) Output Variable*

3. Design and Analysis of the Experiment

In general, for process control, it is expected a closed loop response that provides minimum deviations from the desired operation point and, consequently, minimum tracking error. Therefore, a multilevel factorial experiment was designed and carried out in order to obtain a new set of tuning rules for PI-Fuzzy Logic Controllers based on first order plus dead time process model (FOPDT); taking finto account, the expected closed loop response.

In this factorial experiment, four different factors were chosen in order to obtain the tuning ecuations: the first three of them were the FOPDT model parameters, (a) the process gain K_1, (b) the time constant i (c) and dead time to time constant ratio $t_f$it; the fourth factor was (d) the sample time T included because the FLC is a digital controller and this parameter could affect it performance. The experimental design is know as a 3^4; the three equally spaced levels defined to each factor are given in Table 2. The experimental design was generated by Statgraphics Centurion XIV®. A total of 81 runs without replications or blocks were carried out to complete the experiment. The experimental runs order was completely randomized.

Table 2. *Experimental Design: Factors and Levels*

Level	**Experimental Factors**			
	Ki›	T	t)ir	
Low	0.5	0.5	0.2	0.05
Medium	^{2}.0	3.0	0.6	0.30
High	3.5	5.5	1.0	0.55

The experiment outputs are K_e, K_{de} and K_{dni}, the optimum PI-Fuzzy Logic Controller tuning parameters for every given process model. In each experimental run, a cost function is minimized by varying the set of PI-FLC parameters using Simulink® and Matlab®. The cost function was defined as:

$$O(Ke, K_{dm}) = 11e(t)idt + o^2 c(t) \qquad (1)$$

The experimental results were studied performing an analysis of variance with a confidence interval of 95% using Statgraphics Centurion XIV® software. The influence of the main factors effects and their second order interaction in the PI-FLC tuning parameters were identified. For each case, the normality, constant variance and independence assumption were checked by examining the residual plots and no violations were found. The most significant factors, obtained by the ANOVA analysis, are summarized on Table 3.

Table 3. Results Summary

PI-FLC Tuning Parameters	Experimental Factors			
	K1		tok	
K_e	x	x		x
	x	x	x	x
	X			N

Three regression models were found to fit the experimental data obtained for **K_e, K_{de}** and $K_{dni.}$ Different equations structures were proposed based on the analysis results. The coefficients and exponents associated with significant factors were adjusted in order to obtain the regression model that provides the best coefficient of determination $R^2$for each PI-FLC controller parameter. The proposed set of equations is given in Table 4.

Table 4. PI-Fuzzy Logic Controller Tuning Equations

Equation	Coefficients			
	a			
$Ke = {}_a \left(\frac{T}{K_{pT}} \right)^b$	0.0532	-0.8439		0.8795
$Q_{tlp} = t_o)^{L'} (T)^c$ T	0.1595	0.1885	-0.5195	0.9542
$a = Kd$ KpT^o $(t^o$ T	5.6664	0.0450	0.0571	0.9366

4 MODELING AND VALIDATION RESULTS — CASE STUDY

This section will show the results of simulation tests carried out in order to compare the PI-FLC performance with some reference cases. The controllers were tested in the same process and tuned based on the same process parameters.

CASE STUDY: HOT COLD TANK

Let considered the mixing tank shown in Figure 4. It is necessary to implement a control strategy in order to maintain the mixing temperature at the desired value.

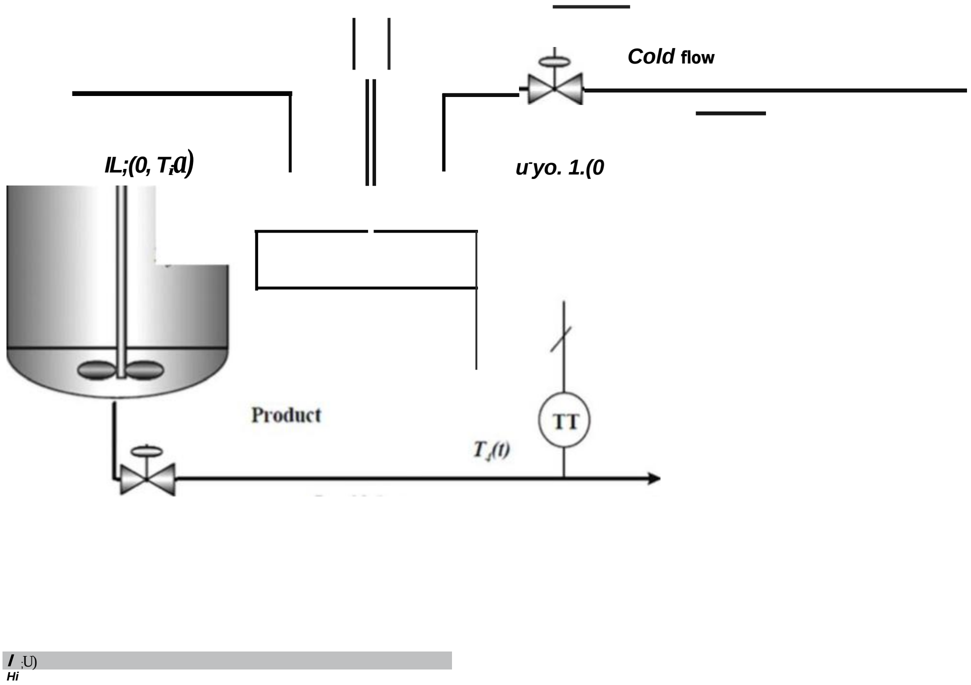


Figure 4. Mixing Tank

The process identification was performed to obtain the FOPDT parameters used to tune the implemented controllers in each case. For a set point change, the dynamic response of the closed loop was plotted and compared in each case. The tuning parameters used are those obtained directly using the corresponding tuning equations without adjusting.

4.1. PID Controller vs. PI-FLC Controllers

The performance of a **PID** controller tuned with no overshoot is compared with the performance of a PI-Fuzzy Logic Controller tuned using the tuning equations developed by Gutierrez and Sanjuan. Additionally, the performance of a PI-

FLC tuned with the proposed tuning equations was also compared. All the controllers were tested with the same process parameters. Results are shown below (see Figure 5).

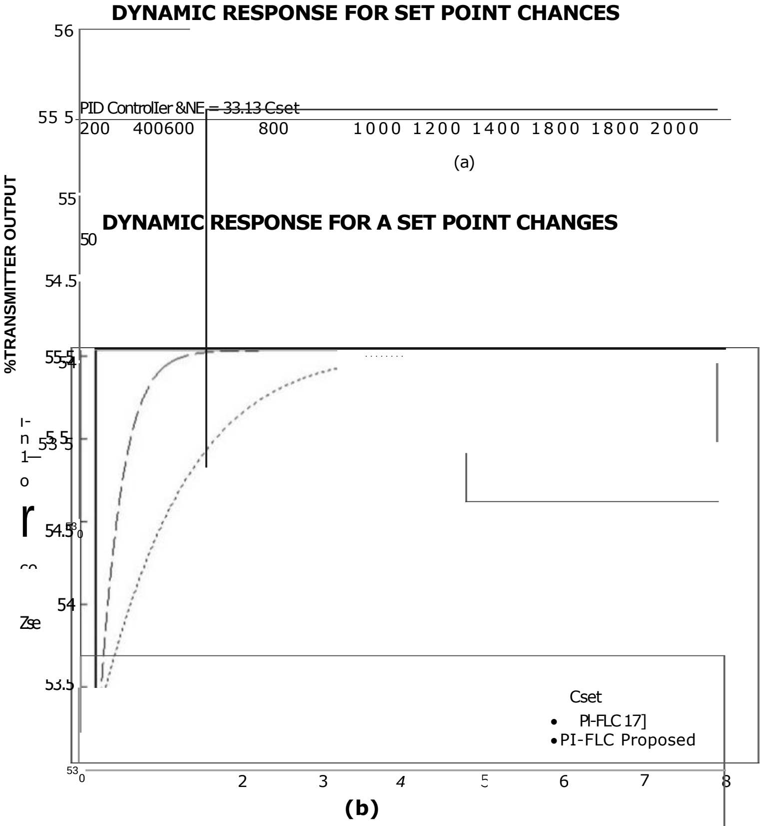


Figure 5. Dynamic Response for (a) PID Controller IAE= 33.13 (b) PI-FLC (Gutierrez & San Juan, 2006) vs. PI-FLC proposed

It was also compared the controller output signal performance in both PI-FLC cases. The results are shown below (see Figure 6).

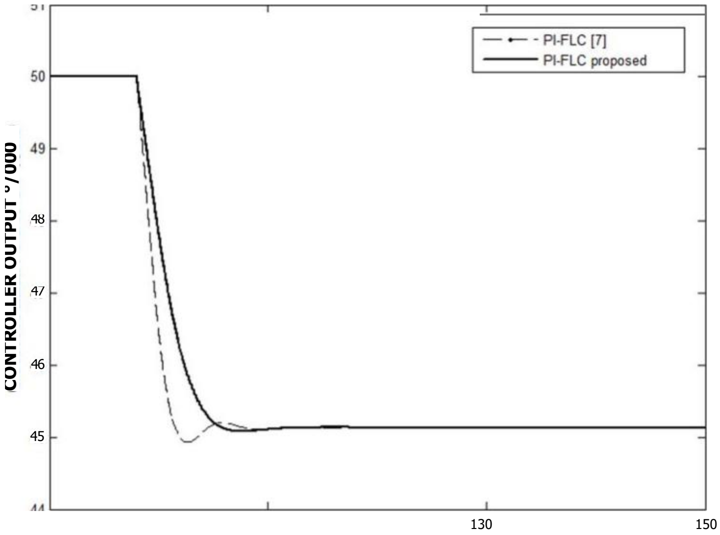


Figure 6. Controller Output Signal Performance (%C0)

4.2. Previous tuning equations (Gutiérrez & Sanjuan, 2006) vs. proposed tuning equations in a simplest FLC.

In this case, it will be compared the performance of a simplest Fuzzy Logic Controller tuned with (Gutiérrez & Sanjuan, 2006) and the same FLC tuned with the equations proposed in the present investigation. A simplest Fuzzy Logic Controller means a FLC with the minimum possible number of membership functions (2) for every fuzzy variable. Both controllers were tested with the same process parameters. These results are shown below (see Figure 7). Although previous tuning equations (Gutiérrez & Sanjuan, 2006) were not developed for this simplified FLC, this tests demonstrates the robustness of the new set of tuning equations to changes in the number of membership functions.

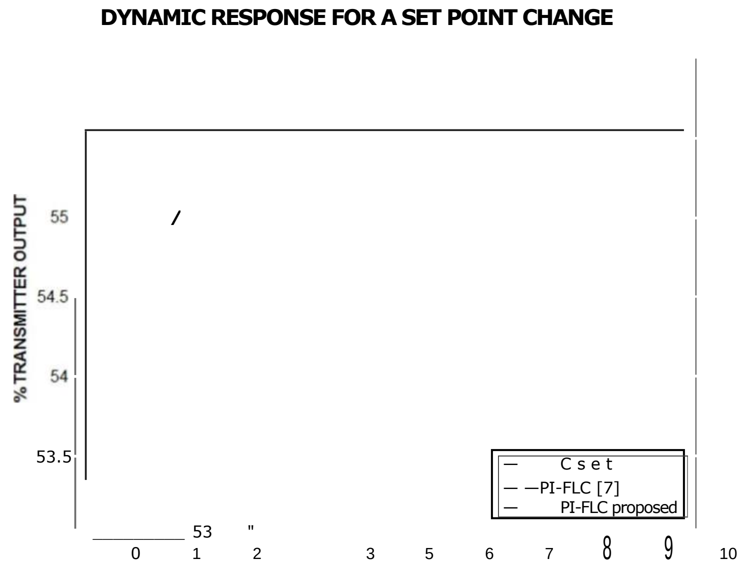


Figure 7. *Dynamic Response for PI-FLC (Gutiérrez & Sanjuan, 2006) and PI-FLC proposed*

5. DISCUSSION AND CONCLUSIONS

In the first case (see Figure 5), it is shown the behavior of the system dynamic response with a PID controller and with a PI-FLC. The IAE for the PI-Fuzzy Logic Controller (28.98) is smaller than the one with the PID controller (33.13), although the difference between both of them is not significant. As the PID controller is easier to implemented, it will be better in this case, unless it cannot control in some process operation conditions. In Figure 5(b) it is shown two process dynamic responses with two PI-FLCs implemented, one of them tuned with the proposed tuning equations and the other one with those developed by Gutierrez and Sanjuan. It can be observed that the process with the PI-FLC tuned with the proposed equations, reaches the steady state before of the same one tuned with the Gutierrez-Sanjuan equations. The values of the Fuzzy Controller gains obtained with the proposed equations are greater so the controller is more aggressive and consequently it has a faster response.

Finally, in the last case (see Figure 7) the simplest PI-FLC tuned with Gutierrez — Sanjuan tuning equations does not reach the steady state

because it is not aggresive.

In addition, it can be conclude that fuzzy Logic Controllers are a powerful tool in process control because they are designed to simulate the operator decision-making ability. In this paper, a tuning equation set are developed and validated, obtaining as a resultan aggressive and fast controller.

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