"CRADLE-TO-GRAVE" SUSTAINABILITY: EXTENSION OF INPUT-OUTPUT MODELS TO MUNICIPAL SOLID WASTES AND TO CORPORATE SOCIAL AND ENVIRONMENTAL RESPONSIBILITY IN THE RETAIL SECTOR[1]

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Introduction

After 21 years, the solid waste act, laid down in August 2010 by the Presidency of Federated Republic of Brazil, establishes, so as to define strategies of a National Policy for Solid Wastes (NPSW): *a*) post-consumption responsibility across industry and retail sector; *b*) targets for reducing waste generation; *c*) defensive measures against environmental damages; *d*) "shared responsibility" among governments, industry, commerce and consumers for direct and indirect generation of wastes (ZANATTA, 2010). Although somewhat indefinite by now, this responsibility will imply, mainly to industries and retailers, logistic and operational costs whose financing and economic incentives the law is not yet clear about (BOURSCHEIT, 2010). Before this legal breach, the best these sectors are left to do is lining up their businesses and growth strategies with a socially and environmentally responsible culture (NAKAMURA; CAMPASSI, 2005).

The first step towards it is linking the net benefits (revenues less private costs or gains less private losses) of firms to the wastes (external costs) their activity produces — or, as in the technical jargon of economics, "internalisation of externalities". Within industrial ecology, it means to follow the "product life cycle" and to ensure sustainability from "cradle" to "grave", so as to say.

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In any event, whether for ecological efficiency or economic reasons, it is widely acknowledged that less polluting practices have won out through economic instruments (taxation and market solutions) rather than through regulation (command-and-control). Most of the times, though, while taxation means rising polluters' financial burden (environmental taxes), pollution markets transfer indirect subsidies to players who can potentially cause great harm to the environment. In the absence of one or another solution, economic inefficiency and ecological ineffectiveness of the environmental legislation (regulations, compliance, prohibitions, standards and fines) prevail.

"In the market of urban solid waste services, the main commodity is a combination of several distinct activities — collection, transport and disposal of urban solid wastes" (THOMAS; CALLAN, 2010, p. 440). On the supply side, private enterprises gather to, under contract with cities, districts and municipalities, deliver services directly to the community. In this case, production costs include the operation of a fleet of trucks for litter collection, the management of a landfill or incineration site and labour costs. On the demand side, are the urban solid waste generators' purchase decisions. Usually, the demand for urban solid waste services goes up with society's income and down with waste generators' environmental liability. The former of these impacts usually arises from economic growth; the latter is measured by the purchases of package-saving commodities (THOMAS; CALLAN, 2010). However, the historical experience has demonstrated that, as economic growth strengthens, the "affluent society" it gives rise to turns into an "effluent society", because of the increasing amount of wastes it generates (DALY, 1968).

By influencing the demand for urban solid waste services, the retail sector stands out for its peculiarity of, at once, generating or reducing wastes at both ends (supply and consumption). On one hand (consumption or output end), the higher the society's income (measured in monetary units, such as, for instance, R$), the larger the quantities of products (measured in physical units, such as, for instance, tonnes) purchased and replaced. On the other hand (supply or input end), the more wastes (measured in physical units, such as, for instance, tonnes) are reduced at source — which, for retailers, is the suppliers chain —, the less the recycling, reuse and collection costs (measured in monetary units, such as, for instance, R$) will be. It is possible, though, to make a choice between generating the same amount of wastes either at a higher cost of recycling and reuse, or at a higher cost — due to a rising demand — of urban solid waste services. The difference, in this case, is that the former strategy is environmentally friendly and compliant with the solid waste act; the latter is not.

Augmented input-output models let determine how much waste (pollution) is needed to yield a monetary unit of economic output or income. Thus, it is possible to figure out what the environmental cost (or benefit) of the economic activity is. The advantage of these models is that they allow to carry out such an evaluation not only across the consumption chain (solid or hazardous wastes), but also across the supply chain (production wastes or scrap) — or, in the case of retail sector, across the suppliers chain (wastes resulting from the stocks of goods for retail sales). Retailers, in particular, are offered the possibility of programming their financial outcomes (net profits) based on targets set for the waste generation, either across the supply or consumption chain.

In the following section, the methodological assumptions underlying hybrid input--output models are presented. Next, the mathematical model to be used is described, and a calculation algorithm for the pollution multipliers is developed. Although these elements depend on the Leontief input inverse matrix or on the Jones (1976) output inverse matrix, the calculation put forward here does not require that the intersectoral matrices of technical coefficients be square. As long as there is a hybrid relationship between inputs measured in physical units and the economic output, measured monetarily, it would not be consistent requiring that the number of biophysical inputs used should equal the number of the existing activity sectors — or, in the case of the retail sector, consumption categories. This would amount to an unrealistic requirement, just to meet the algebraic feature that makes it possible to invert square matrices only. In the third section, the method is applied to a hypothetical retail sector, so as to gauge the monetary impacts arising from a reduced use of biophysical inputs — and vice-versa. Then, after the discussion of the results, conclusions are drawn.

Methodology

Assumptions

In the late 1960's, economic input-output analysis started being applied to energy sectors and to environmental problems (MACHADO *et al.*, 2001). Originally, the Leontief input-output model (cell (2) of Table 1 and Table 2; Table 3) had been proved useful to cope with the interdependence phenomenon, once it managed to bring out how the annual flow of economic income or output (*final* goods and services) was actually supported by an *intermediate* flow of commodities. Next, by analogy, the same reasoning was extended to argue that the flow of all economic commodities (both final and intermediate ones) was also sustained by *physical* flows (cells (1) and (3) of Table 1 and Table 2) that, in spite of bearing no price whatsoever, did serve as an indispensable complement of the monetary flows (cells (2) of Table 1 and Table 2) of goods and services (DALY, 1968).

Table 1–Augmented input-output model

From	To	
	Human	Non-human
Human	(2) economy – economy economics	(1) economy – nature (waste sinks)
Non-human	(3) nature – economy (environmental sources)	(4) nature – nature ecology

Sources: DALY (1968, p. 401); PERMAN *et al.* (1996, p. 13)

Table 2–Unfolding of the augmented input-output model (Table 1)

Inputs (from)	Outputs (to)								Total
	Activity sectors								
	Transforming sectors		Primary	Transforming sectors				Primary	
	Living	Non-living	Final consump	Living	Non-living			Final consump	
	Agricult. (1)	Industry (2)	House-holds (3)	Biosph. (4)	Atmosph (5)	Hidrosph (6)	Lithosph (7)	Thermodyn sinks (8)	
	Quadrant (2)			**Quadrant (1)**					
1. Agric.	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}	y_{16}	y_{17}	y_{18}	Y_1
2. Ind.	y_{21}	y_{22}	y_{23}	y_{24}	y_{25}	y_{26}	y_{27}	y_{28}	Y_2
3.Househ. (primary services)	y_{31}	y_{32}	y_{33}	y_{34}	y_{35}	y_{36}	y_{37}	y_{38}	Y_3
	Quadrant (3)			**Quadrant (4)**					
4. Biosph.	y_{41}	y_{42}	y_{43}	y_{44}	y_{45}	y_{46}	y_{47}	y_{48}	Y_4
5. Atmosph.	y_{51}	y_{52}	y_{53}	y_{54}	y_{55}	y_{56}	y_{57}	y_{58}	Y_5
6. Hidrosph.	y_{61}	y_{62}	y_{63}	y_{64}	y_{65}	y_{66}	y_{67}	y_{68}	Y_6
7. Lithosph.	y_{71}	y_{72}	y_{73}	y_{74}	y_{75}	y_{76}	y_{77}	y_{78}	Y_7
8. Sun (primary services)	y_{81}	y_{82}	y_{83}	y_{84}	y_{85}	y_{86}	y_{87}	y_{88}	Y_8

Source: DALY (1968, p. 402)

This augmented model (Tables 1 and 2) shows that cell (1) is connected to cell (3) by cell (4) and that cell (3) directly influences human well-being (income and consumption levels). In Table 2, every entry (y_{ij}) in cell (2) represents "economic commodities", whereas, in the remaining cells (1, 3 and 4), these entries stand for "ecological commodities" and set the "biophysical foundations of economics" (DALY, 1968).

Traditionally, economics has been exclusively concerned with cell (2) (Tables 1 and 2) — more thoroughly detailed by Table 3. For simplicity and saving of space, the geometric array displayed in Table 3 is used to be shortened by matrix algebra (Eqs. (2) and (3)).

In Table 3, the basic underlying assumption is that the intermediate demand y_{ij} per activity sector A_1, ..., A_n keeps a relatively unchangeable proportion with the economy's total output (Y_i) or income (Y_j) (Eqs. (1), (2) and (4)). If it is so indeed, it is possible, as in Eq. (1), to convert every intermediate expenditure (y_{ij}) of the economic activity into proportions (percentages) of the total monetary income (payments) (Y_j) or of the monetary value of the total economic output (Y_i), since, in accountancy, the expenditures on the economic output demanded must be theoretically equal to the ability to pay for it (income).

Table 3–Standard input-output model (Leontief model)

	Activity sectors				
	Intermediate demand			Final demand †	Total
Outputs (j) Inputs (i)	A_1	…	A_n	F_i	Y_i
A_1	y_{11}	…	y_{1n}	F_1	Y_1
⋮	⋮		⋮	⋮	⋮
A_n	y_{n1}	…	y_{nn}	F_n	Y_n
P_j^*	P_1	…	P_n	$P = F$	
Y_j	Y_1	…	Y_n		Y

(†) $F = C$ (consumption) + I (investment) + G (government expenditures) + X (exports).
(*) P_j = payments to production factors or economy's incomes; usually includes wages, profits and imports.

$$a_{ij} = \frac{y_{ij}}{Y_j} \tag{1}$$

Horizontal reading of Table 3

$$\mathbf{A}_{i\times j}\mathbf{Y}_j + \mathbf{F}_i = \mathbf{Y}_i \quad \rightarrow \text{with } \mathbf{Y}_i = \mathbf{Y}_j \tag{2}$$

$$\mathbf{Y}_j = \underbrace{(\mathbf{I} - \mathbf{A})^{-1}}_{j\times i}\mathbf{F}_i \tag{3}$$

Vertical reading of Table 3

$$\mathbf{Y}_i\mathbf{A}_{i\times j} + \mathbf{P}_j = \mathbf{Y}_j \quad \rightarrow \text{with } \mathbf{Y}_i = \mathbf{Y}_j \tag{4}$$

$$\mathbf{Y}_j = \mathbf{P}_j\underbrace{(\mathbf{I} - \mathbf{A})^{-1}}_{j\times i} \tag{5}$$

where I = identity matrix and $\mathbf{A}_{i\times j}$ = square matrix, with $i = j$.

These ratios, defined by Eq. (1), are called *technical coefficients* of the Leontief input--output model (Table 3). The relationships between monetary expenditures (income) and gains (output) they express describe, at certain point, the "state of the art" (technique) existing in a given production system. However, insofar as the Leontief model is augmented to include the "ecological commodities" of cells (1), (3) and (4) in Tables 1 and 2, the coefficients a_{ij} represent biophysical rather than monetary quantities. For example, if one pound of alfalfa takes 900 pounds of water to be produced, then a_{ij} = 900. In this case, the entries a_{ij} are called *natural technical coefficients* (DALY, 1968) and usually take on values greater than unity.

As regards macroeconomics, monetary accounts and bookkeeping, income spent must not outstrip earnings arising from output sales. Therefore standard technical coefficients are necessarily positive and smaller than unity. Biophysically though, the final output represents no more than a fraction of all matter and energy used up to produce it. The remaining is wasted matter and energy, which although have not disappeared, are economically unavailable, because of the high entropy state in which they can thermodynamically be found. Thus, since the biophysical "cost" of material output is thermodynamically larger than gains, *natural* technical coefficients use to be greater than unity.

The discrepancy between the values of natural technical coefficients and those of the Leontief ones owes to the strictly monetary treatment provided by the standard input-output model (Table 3 and cell (2) of Tables 1 and 2). Because of this reductionism – which boils down "everything to its monetary aspect" (SÖDERBAUM, 2008, p. 2) –, the benefits obtained from production and consumption of economic goods and services are split up from the bads and disservices that economic activity causes not only to ecosystems, but also to itself. The unwanted inputs (pollutants and contaminants) generated by production processes find no counterpart in monetarily measured economic transactions (AYRES; KNEESE, 1969).

The basic assumption underlying the augmented input-output model is adding to monetarily gauged inputs (rows of Table 3) other inputs measured in common physical units – for instance, tonnes of carbon equivalent (CO2eq), tonnes of oil equivalent (toe), tonnes (t), cubic metres (m3), hectares (ha), square kilometres (km2), parts per million (ppm) etc. (MACHADO *et al.*, 2001; BUTTNAR; LLOP, 2007). To accomplish that, sub-matrices are added above (matrix **N** of sources of natural resources — raw materials and energy) and below (matrix **W** of production and consumption *wastes*) Table 3 (PERMAN *et al.*, 1996; MILLER; BLAIR, 2009). One and another record the *throughput* (DALY, 1974; Eqs. (6) e (7)) — the ecological cost of maintaining and replenishing the stocks of economic commodities — needed to support the periodical service flows (benefits) sprung from the economic activity (production and consumption).

$$\overbrace{\text{Ultimate efficiency}}^{(1)} = \overbrace{\text{Service efficiency}}^{(2)} \times \overbrace{\text{Maintenance efficiency}}^{(3)} \quad (6)$$

$$\frac{\text{Service}}{Throughput} = \frac{\text{Service}}{\text{Stock}} \times \frac{\text{Stock}}{Throughput} \quad (7)$$

Sub-matrix **N** records the use of *i* natural resources, in weight or volume units (*e.g.*, tonnes), per *j* activity sector – or, in the case of the retail sector, per *j* consumption category (*e.g.*, food, hygiene and cleansing, electro-electronics etc.). Likewise, sub-matrix **W** includes the quantity, in weight or volume units, of *i* wastes per *j* activity sector – or, in the case of the retail sector, per *j* consumption category. Nonetheless, sub-matrices **N** and **W** hold *i* inputs which are expressed in physical units (*e.g.*, tonnes), whereas the *j* activity sectors (or consumption categories) yield monetarily measured benefits (in $, as in matrix **Y**, of Eqs. (2) through (5)). Thus, in order to link physical units in matrices **N** and **W** to monetary units in matrix **Y**, it is needed, in either case, to find a matrix of *hybrid* technical coefficients (**B**, in Eq. (8), e **V**, na Eq. (9)) that communicates, through an invariable ratio, the relationship between physical quantities of inputs (*e.g.*, in tonnes, *t*) and monetary units of output (in $)[i].

$$\underbrace{\mathbf{N}_i}_{\text{t}} = \underbrace{\mathbf{B}_{i\times j}}_{\frac{\text{t}}{\$}} \times \underbrace{\mathbf{Y}_j}_{\$} \tag{8}$$

$$\underbrace{\mathbf{W}_i}_{\text{t}} = \underbrace{\mathbf{V}_{i\times j}}_{\frac{\text{t}}{\$}} \times \underbrace{\mathbf{Y}_j}_{\$} \tag{9}$$

By substituting Eq. (3) into Eqs. (8) e (9), it turns out:

$$\underbrace{\mathbf{N}_i}_{\text{t}} = \underbrace{\mathbf{B}_{i\times j}}_{\frac{\text{t}}{\$}} \times \underbrace{\left[(\mathbf{I}-\mathbf{A})^{-1}_{j\times i}\mathbf{F}_i\right]_j}_{\$} \tag{10}$$

$$\underbrace{\mathbf{W}_i}_{\text{t}} = \underbrace{\mathbf{V}_{i\times j}}_{\frac{\text{t}}{\$}} \times \underbrace{\left[(\mathbf{I}-\mathbf{A})^{-1}_{j\times i}\mathbf{F}_i\right]_j}_{\$} \tag{11}$$

Unlike matrix **A** (Eq. (2)), the *hybrid* matrices **B** (Eqs. (8) and (10)) and **V** (Eqs. (9) and (11)) are not necessarily square ($i \neq j$), since neither the classes of natural resources nor those of polluting residues need to coincide with the number of economic sectors or categories that demand them.

As Table 4, below, indicates, the hybrid matrix **B** leads to cell (3), in Tables 1 and 2, whereas the hybrid matrix **V** corresponds to the transposition of the array displayed in cell (1), in those tables.

Table 4– Matrix transcription of the augmented input-output model (Tables 1 and 2)

Outputs (*j*) / Inputs (*i*)	Economy	Ecology	Total
Economy	**(2)** **A** ($/$)	**(1)** **V'** ($/t)*	**Y** ($)
Ecology	**(3)** **B** (t/$)	**(4)** .... (t/t)	**W** (t)
Total	**Y** ($)	**N** (t)	

Source: By the author.

(*) The superscript (') stands for matrix transposition. Although this operation is algebraically needless in Eqs. (9) and (11), the transposition, in quadrant (1) of Table 4, is but a symbolic trick to bring consistency to the comparison between Tables 1, 2 and 4.

Both in Eq. (10) and in Eq. (11), the multiplication indicated on the right hand side of the equations represents the *multiplier effect of pollution*, which measures the variation in the quantity of the pollutant of kind i ($d\mathbf{N}$ ou $d\mathbf{W}$) generated by a unit and exogenous change in the final demand of the activity sector (BUTNAR; LLOP, 2007) or consumption category j ($d\mathbf{F}$) (Eqs. (12)*a-b* e (13)*a-b*). Unlike Eqs. (8) e (9), as Eqs. (10) e (11), as noticed, do not directly depend on **Y** — economic income (turnover) or output; they only depend on the final demand **F**. However, by Table 3 and Eq. (3), it can be verified that, indirectly, changes in **F** cause **Y** to vary. This means that, even though the growth of the economy, income and turnover (Δ**Y**) also determines, through standard macroeconomic[ii] multipliers, the expansion of **F**, the multiplier effect of pollution is mainly caused by the pressures on the final demand. For example, a rise or fall in aggregated consumption (C), investment (*I*) and/or exports (*X*) can affect the quantity of wastes generated. The sensibility of waste throw into the economy in response to changes in the final demand (**F**) is given by Eqs. (12)*a*, (12)*b*, (13)*a* e (13)*b*, which represent the *pollution multipliers* (BUTNAR; LLOP, 2007).

$$d\mathbf{N} = \left[\mathbf{B}(\mathbf{I} - \mathbf{A})^{-1}\right]d\mathbf{F} \qquad (12)a$$

$$\frac{d\mathbf{N}}{d\mathbf{F}} = \mathbf{B}(\mathbf{I} - \mathbf{A})^{-1} \qquad (12)b$$

$$d\mathbf{W} = \left[\mathbf{V}(\mathbf{I} - \mathbf{A})^{-1}\right]d\mathbf{F} \qquad (13)a$$

$$\frac{d\mathbf{W}}{d\mathbf{F}} = \mathbf{V}(\mathbf{I} - \mathbf{A})^{-1} \qquad (13)b$$

By means of pollution multipliers (Eqs. (12)*a*, (12)*b*, (13)*a* e (13)*b*), the input-output models can explain, in depth, the process of waste generation. In particular, it can be analysed how changes in the final demand alter the percentage composition of municipal solid wastes (environmental cost) in the economic income (turnover) or output (BUTNAR; LLOP, 2007).

Mathematical model and calculation methodology

Next, a calculation procedure is described, which allows, from pollution multipliers, to arrive at the matrix of *intensity coefficients* (MACHADO *et al.*, 2001, p. 413) of wastes per *j* activity sector. This matrix actually corresponds to the *throughput* or *biophysical cost* of economic production (in t/$). It is then possible, from the reciprocal of its entries, to get a new matrix that informs the *monetary cost* of wastes per *j* activity sector (in $/t).

In the retail sector case, it is suitable to replace the *j* activity sectors in the standard model by *j* "consumption categories" (matrices (19) and (20)). After this adjustment, matrix **A**, which describes intersectoral demands, represents the retailers' (columns) monetary expenditures with suppliers (rows) per consumption category.

In matrix (19), entries *in* the main diagonal, where $i = j$, should be interpreted, *directly*, as supplies to make up the retailers' stocks of the corresponding category; entries *off* the main diagonal, where $i \neq j$, should be thought of as the *indirect* impacts that the rise of the turnover at a given consumption category causes onto the (intermediate) demand for the supplies of another's (JONES, 1976). For instance, as the retailers' turnover on food commodities rises, it is likely that part of that increment is used to purchase more hygiene and cleansing[iii], whose value added per unit sold is certainly greater.

A simple way of estimating these substitutions and increments is through price--elasticity (Eq. (14)) and income-elasticity (Eq. (15)) of demand per consumption category. The price-elasticity of demand allows for estimating turnover changes caused by price (*P*) changes at any *j* consumption category (*Q*). In addition, the income-elasticity of demand allows foreseeing expenditure (*Q*) changes per *j* consumption category, as the retailers' income (turnover) (*Y*) shifts.

$$\epsilon_P = -\frac{dQ_j}{dP_j}\frac{P_j}{Q_j} \quad (14)$$

$$\epsilon_Y = \frac{dQ_j}{dY_j}\frac{Y_j}{Q_j} \quad (15)$$

The hybrid version of matrix (19) is provided by Table 5 to fit for the retail sector. For a complete treatment of sustainability, hybridism should occur both in the supply chain (input side) and in the consumption chain (output side). In the supply chain, natural resources, which make up matrix **N** (16) of the products supplied for retail, are included – i rows of matrix **B** (18); in the consumption chain, wastes, accounted of by matrix **W** (24), are brought in – i rows of matrix **V** (23).

The components of the consumption categories are measured in monetary units ($), while the components of matrices **N** and **W**, are measured in physical weight units (tonnes, t). The different orders of matrices **N** (4 x 1), **A** (3 x 3) and **W** (2 x 1) ((16), (20) and (24)) are deliberately chosen to demonstrate that there is no need for coincidence between the number of natural resources, economic activity sectors or consumption categories and the considered wastes.

Table 5–Hybrid composition of a hypothetical input-output model applied to retail

Outputs *j* / Inputs *i*	Consumption categories (*j*)			Total
Matrix **N** of *i* natural resources	1) Food	2) Hygiene and Cleansing	3) Electronics	(tonnes, t)
1) Metals	r_{11}	r_{12}	r_{13}	N_1
2) Water	r_{21}	r_{22}	r_{23}	N_2
3) Cellulose	r_{31}	r_{32}	r_{33}	N_3
4) Oil	r_{41}	r_{42}	r_{43}	N_4
Total ($)	Y_1	Y_2	Y_3	
Matrix **W** of *i* wastes	1) Food	2) Hygiene and Cleansing	3) Electronics	Total (tonnes, t)
1) Paper	u_{11}	u_{12}	u_{13}	W_1
2) Plastic	u_{21}	u_{22}	u_{23}	W_2
Total ($)	Y_1	Y_2	Y_3	

Source: By the author.

$$\underbrace{\mathbf{N}_{4\times 1}}_{\text{t}} = \begin{bmatrix} \text{metals} & N_1 \\ \text{water} & N_2 \\ \text{cellulose} & N_3 \\ \text{oil} & N_4 \end{bmatrix} \tag{16}$$

$$\underbrace{\mathbf{R}_{4\times 3}}_{\text{t}} = \begin{bmatrix} \text{(t)} & \text{food} & \text{hyg/clean.} & \text{electr.} \\ \text{metals} & r_{11} & r_{12} & r_{13} \\ \text{water} & r_{21} & r_{22} & r_{23} \\ \text{cellul.} & r_{31} & r_{32} & r_{33} \\ \text{oil} & r_{41} & r_{42} & r_{43} \end{bmatrix} \tag{17}$$

$$\underbrace{\mathbf{B}_{4\times 3}}_{\text{t/\$}} = \begin{bmatrix} \text{(t/\$)} & \text{food} & \text{hyg/clean.} & \text{electr.} \\ \text{metals} & b_{11} & b_{12} & b_{13} \\ \text{water} & b_{21} & b_{22} & b_{23} \\ \text{cellul.} & b_{31} & b_{32} & b_{33} \\ \text{oil} & b_{41} & b_{42} & b_{43} \end{bmatrix} \tag{18}$$

$$\underbrace{\textbf{Intermediate Demand}_{3\times 3}}_{\$} = \begin{bmatrix} \text{(\$)} & \text{food} & \text{hyg/clean.} & \text{eledtr.} \\ \text{food} & y_{11} & y_{12} & y_{13} \\ \text{hyg/clean.} & y_{21} & y_{22} & y_{23} \\ \text{electr.} & y_{31} & y_{32} & y_{33} \end{bmatrix} \tag{19}$$

$$\underbrace{\mathbf{A}_{3\times 3}}_{\$/\$} = \begin{bmatrix} \text{(\$/\$)} & \text{food} & \text{hyg/clean.} & \text{electr.} \\ \text{food.} & a_{11} & a_{12} & a_{13} \\ \text{hyg/clean.} & a_{21} & a_{22} & a_{23} \\ \text{electr.} & a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{20}$$

$$\underbrace{\mathbf{Y}_{3\times 1}}_{\$} = \begin{bmatrix} \text{food} & Y_1 \\ \text{hyg/clean.} & Y_2 \\ \text{electr.} & Y_3 \end{bmatrix} \tag{21}$$

$$\underbrace{\mathbf{U}_{2\times 3}}_{\text{t}} = \begin{bmatrix} \text{(t)} & \text{food} & \text{hyg/clean.} & \text{electr.} \\ \text{paper} & u_{11} & u_{12} & u_{13} \\ \text{plast.} & u_{21} & u_{22} & u_{23} \end{bmatrix} \tag{22}$$

$$\underbrace{\mathbf{V}_{2\times 3}}_{\text{t/\$}} = \begin{bmatrix} \text{(t/\$)} & \text{food} & \text{hyg/clean.} & \text{electr.} \\ \text{paper} & v_{11} & v_{12} & v_{13} \\ \text{plast.} & v_{21} & v_{22} & v_{23} \end{bmatrix} \tag{23}$$

$$\underbrace{\mathbf{W}_{2\times 1}}_{\text{t}} = \begin{bmatrix} \text{paper} & W_{\mathbf{1}} \\ \text{plast.} & W_2 \end{bmatrix} \tag{24}$$

Matrices **R** (17) and **U** (22) record, respectively, *i* material (or energy) requirements and *i* wastes observed in each *j* category of retail consumption or economic activity sector (Table 5). Both matrices **R** ((17)) and **U** ((22)) contain elements that are measured in physical units of weight (tonnes, t) or volume. Based on elementary operations indicated by Eqs. (25) and (26) below, the hybrid matrices **B** ((18)) and **V** ((23)) are respectively arrived at, the elements of which being expressed by a ratio of physical to monetary units (t/$).

The hybrid matrices **B** ((18)) and **V** ((23)) contain the so-called *natural technical coefficients*, as the non-hybrid matrix **A** ((20)) contains the *pure technical coefficients*, calculated through Eq. (1), from matrices (19) and (21). Similarly to the pure technical coefficients, the *natural technical coefficients*, contained in matrices **B** (18) and **V** (23), describe the current patterns or technological processes, this time determined, however, by the material requirements and by the quantity of residues implied by the economic activity.

$$b_{ij} = \frac{r_{ij}}{Y_j} \tag{25}$$

$$v_{ij} = \frac{u_{ij}}{Y_j} \tag{26}$$

A sequence of elementary algebraic operations with matrices reveals the impact that this production technology implies not only to the demand for raw material *sources* and *sinks* of waste assimilation, but also to the monetary costs of the economic activity. This calculation algorithm is described below (Eqs. (27) to (37)):

Table 6–Throughput calculation in the retail sector

Description	Input end – Wastes contained in stocks: Supply chain (suppliers) – Operation	Eq.	Output end – Consumption wastes: Consumption chain – Operation	Eq.
1) MSW* pollution multiplier effect	$\mathbf{M} = \frac{d\mathbf{N}}{d\mathbf{F}}\mathbf{Y} \rightarrow$ $\underbrace{\mathbf{M}}_{\substack{t \\ (4\times1)}} = \left[\underbrace{\mathbf{B}_{(4\times3)}}_{\frac{t}{\$}}\underbrace{(\mathbf{I}-\mathbf{A})^{-1}_{(3\times3)}}_{\frac{\$}{\$}}\right]\underbrace{\mathbf{Y}_{(3\times1)}}_{\$}$	(27) *a*	$\mathbf{S} = \frac{d\mathbf{W}}{d\mathbf{F}}\mathbf{Y} \rightarrow$ $\underbrace{\mathbf{S}}_{\substack{t \\ (2\times1)}} = \left[\underbrace{\mathbf{V}_{(2\times3)}}_{\frac{t}{\$}}\underbrace{(\mathbf{I}-\mathbf{A})^{-1}_{(3\times3)}}_{\frac{\$}{\$}}\right]\underbrace{\mathbf{Y}_{(3\times1)}}_{\$}$	(27) *b*
2) Diagonal matrix of the MSW* load due to the pollution multiplier effect	$\widehat{\mathbf{M}} = \begin{bmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{22} & 0 & 0 \\ 0 & 0 & m_{33} & 0 \\ 0 & 0 & 0 & m_{44} \end{bmatrix}$	(28) *a*	$\widehat{\mathbf{S}} = \begin{bmatrix} s_{11} & 0 \\ 0 & s_{22} \end{bmatrix}$	(28) *b*
3) MSW* load (in t) *per monetary unit* of economic output (1$)	$\underbrace{\widehat{\mathbf{M}}^{-1}}_{\frac{1}{t}} = \begin{bmatrix} \frac{1}{m_{11}} & 0 & 0 & 0 \\ 0 & \frac{1}{m_{22}} & 0 & 0 \\ 0 & 0 & \frac{1}{m_{33}} & 0 \\ 0 & 0 & 0 & \frac{1}{m_{44}} \end{bmatrix}$	(29) *a*	$\underbrace{\widehat{\mathbf{S}}^{-1}}_{\frac{1}{t}} = \begin{bmatrix} \frac{1}{s_{11}} & 0 \\ 0 & \frac{1}{s_{22}} \end{bmatrix}$	(29) *b*
4) Monetary cost ($) per unit of MSW* (1t)	$\underbrace{\mathbf{X}}_{\frac{1}{\$}} = \underbrace{\widehat{\mathbf{M}}^{-1}}_{\frac{1}{t}}\underbrace{\mathbf{B}}_{\frac{t}{\$}}$	(30) *a*	$\underbrace{\mathbf{H}}_{\frac{1}{\$}} = \underbrace{\widehat{\mathbf{S}}^{-1}}_{\frac{1}{t}}\underbrace{\mathbf{V}}_{\frac{t}{\$}}$	(30) *b*
5) Proportion of the monetary cost (**C**) in MSW* quantities (in t) or total monetary value ($) of the MSW in the total monetary benefit (**B**) or revenue ($) ≈ **Ratio C/B**	$\mathbf{L} = \underbrace{\mathbf{X}}_{\frac{1}{\$}}\underbrace{\mathbf{Y}}_{\$}$ $\mathbf{L} = \frac{1t\ \times\ \$}{\$} = \frac{\$}{\$}$ $\mathbf{L} = \frac{\text{Total value or monetary cost (in \\$) of the MSW load (in t)}}{\text{Total monetary benefit (in \\$)resulting from the economic output}}$	(31) *a*	$\mathbf{G} = \underbrace{\mathbf{H}}_{\frac{1}{\$}}\underbrace{\mathbf{Y}}_{\$}$ $\mathbf{G} = \frac{1t\ \times\ \$}{\$} = \frac{\$}{\$}$ $\mathbf{G} = \frac{\text{Total value or monetary cost (in \\$) of the MSW load (in t)}}{\text{Total monetary benefit (in \\$)resulting from the economic output}}$	(31) *b*

Table 6–Throughput calculation in the retail sector (continued)

Description	Input end – Wastes contained in stocks – Supply chain (suppliers) – Operation	Eq.	Output end – Consumption wastes – Consumption chain – Operation	Eq.
6) Diagonal matrix with the reciprocal of Eq. (31). ≈ **Ratio B/C**	$\hat{\mathbf{L}}^{-1} = \begin{bmatrix} \frac{1}{\ell_{11}} & 0 & 0 & 0 \\ 0 & \frac{1}{\ell_{22}} & 0 & 0 \\ 0 & 0 & \frac{1}{\ell_{33}} & 0 \\ 0 & 0 & 0 & \frac{1}{\ell_{44}} \end{bmatrix}$	(32) *a*	$\hat{\mathbf{G}}^{-1} = \begin{bmatrix} \frac{1}{g_{11}} & 0 \\ 0 & \frac{1}{g_{22}} \end{bmatrix}$	(32) *b*
7) Matrix of the MSW* *intensity coefficients* or **biophysical cost** (*throughput*) of the economic output (in t/$)	$\underbrace{\mathbf{Z}}_{\frac{t}{\$}} = \left(\hat{\mathbf{L}}^{-1} \underbrace{\mathbf{B}}_{\frac{t}{\$}} \right)$ $\underbrace{\mathbf{Z}_{4\times 3}}_{\frac{t}{\$}} = \begin{bmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \\ z_{41} & z_{42} & z_{43} \end{bmatrix}$	(33) *a*	$\underbrace{\mathbf{E}}_{\frac{t}{\$}} = \left(\hat{\mathbf{G}}^{-1} \underbrace{\mathbf{V}}_{\frac{t}{\$}} \right)$ $\underbrace{\mathbf{E}_{2\times 3}}_{\frac{t}{\$}} = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \end{bmatrix}$	(33) *b*
8) Total MSW* load (in t)	$\underbrace{\mathbf{M}}_{t} = \underbrace{\mathbf{Z}}_{\frac{t}{\$}} \underbrace{\mathbf{Y}}_{\$}$	(34) *a*	$\underbrace{\mathbf{S}}_{t} = \underbrace{\mathbf{E}}_{\frac{t}{\$}} \underbrace{\mathbf{Y}}_{\$}$	(34) *b*
9) **Monetary cost** of the intensive use of MSW* in the economic output (in $/t)	$\underbrace{\mathbf{Z}^*_{4\times 3}}_{\frac{\$}{t}} = \begin{bmatrix} \frac{1}{z_{11}} & \frac{1}{z_{12}} & \frac{1}{z_{13}} \\ \frac{1}{z_{21}} & \frac{1}{z_{22}} & \frac{1}{z_{23}} \\ \frac{1}{z_{31}} & \frac{1}{z_{32}} & \frac{1}{z_{33}} \\ \frac{1}{z_{41}} & \frac{1}{z_{42}} & \frac{1}{z_{43}} \end{bmatrix}$	(35) *a*	$\underbrace{\mathbf{E}^*_{4\times 3}}_{\frac{\$}{t}} = \begin{bmatrix} \frac{1}{e_{11}} & \frac{1}{e_{12}} & \frac{1}{e_{13}} \\ \frac{1}{e_{21}} & \frac{1}{e_{22}} & \frac{1}{e_{23}} \end{bmatrix}$	(35) *b*
10) *Total* monetary cost ($) per *j* sector due to the MSW* load in the economic output	$\underbrace{\mathbf{C}}_{\$} = \underbrace{\mathbf{M'}}_{t} \underbrace{\mathbf{Z}^*}_{\frac{\$}{t}}$	(36) *a*	$\underbrace{\mathbf{K}}_{\$} = \underbrace{\mathbf{S'}}_{t} \underbrace{\mathbf{E}^*}_{\frac{\$}{t}}$	(36) *b*
	$\underbrace{\mathbf{M}_{4\times 1}}_{t} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} \rightarrow$ $\underbrace{\mathbf{M}'_{1\times 4}}_{t} = [m_1 \quad m_2 \quad m_3 \quad m_4]$	(37) *a*	$\underbrace{\mathbf{S}_{2\times 1}}_{t} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \rightarrow$ $\underbrace{\mathbf{S}'_{1\times 2}}_{t} = [s_1 \quad s_2]$	(37) *b*

Source: By the author.
(*) Municipal Solid Wastes = urban solid wastes.

Hypothetical application of the augmented input-output model

In this section, values are hypothetically ascribed to the calculation model described in Table 6, so as to test it and evaluate its results. Although, according to Table 5 and Eqs. (16) to (24), the exercise refers to retail, the calculation methodology theoretically applies, with the due adjustments, to any activity sector.

As announced in the Introduction, the realism that hybridism requires to match biophysical inputs – measured in physical units of weight or volume – and economic outputs – measured in monetary units – does not admit, except by coincidence, that the quantity of economic activity sectors (or consumption categories) be necessarily the same as the number of material and energy inputs (matrix **N** (16)) or of wastes (matrix **W** (24)) involved in the production of the economic output. Through vector diagonalization (Eqs. (28)*a-b*, (29)*a-b* and (32)*a-b*, in Table 6), well known in matrix algebra (MILLER; BLAIR, 2009; MACHADO et al., 2001), it is possible to lend realism to the augmented model.

Table 7–Numerical example and operations in the standard model (Table 3)

Basic operations	Eq.
$\underbrace{\begin{bmatrix} 20 & 25 & 45 \\ 40 & 30 & 50 \\ 15 & 50 & 55 \end{bmatrix}}_{\substack{\text{Intermediate} \\ \text{demand} \\ (3 \times 3)}} + \underbrace{\begin{bmatrix} 10 \\ 30 \\ 80 \end{bmatrix}}_{\substack{\mathbf{F} \\ (3\times1)}} = \underbrace{\begin{bmatrix} 100 \\ 150 \\ 200 \end{bmatrix}}_{\substack{\mathbf{Y} \\ (3\times1)}}$	(2)
$\mathbf{A}_{(3 \times 3)} = \begin{bmatrix} 0.20 & 0.17 & 0.23 \\ 0.40 & 0.20 & 0.25 \\ 0.15 & 0.33 & 0.28 \end{bmatrix}$	(1), (20)
$(\mathbf{I} - \mathbf{A})_{(3 \times 3)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.20 & 0.17 & 0.23 \\ 0.40 & 0.20 & 0.25 \\ 0.15 & 0.33 & 0.28 \end{bmatrix} = \begin{bmatrix} 0.80 & -0.17 & -0.23 \\ -0.40 & 0.80 & -0.25 \\ -0.15 & -0.33 & 0.72 \end{bmatrix}$	
$(\mathbf{I} - \mathbf{A})^{-\mathbf{1}}_{(3\times3)} = \begin{bmatrix} 1.74 & 0.69 & 0.78 \\ 1.15 & 1.91 & 1.01 \\ 0.89 & 1.02 & 2.01 \end{bmatrix}$	

Table 8–Numerical example with hybridism (Table 5) for throughput calculation in the retail sector (Table 6)

Step (Tab. 6)	Input end **Wastes contained in stocks** Supply chain (suppliers) Operation	Eq.	Output end **Consumption wastes** Consumption chain Operation	Eq.
	$\underbrace{\begin{bmatrix} \text{Food} & \text{Hyg.} & \text{Electr.} \\ 150 & 275 & 225 \\ 300 & 250 & 450 \\ 120 & 500 & 230 \\ 200 & 400 & 600 \end{bmatrix}}_{\substack{\mathbf{R}\,(4\times3) \\ (t)}} = \underbrace{\begin{bmatrix} \text{Metals} & 650 \\ \text{Water} & 1000 \\ \text{Cellul.} & 850 \\ \text{Oil} & 1200 \end{bmatrix}}_{\substack{\mathbf{N}\,(4\times1) \\ (\text{t})}}$	(17) and (16)	$\underbrace{\begin{bmatrix} \text{Food} & \text{Hyg.} & \text{Electr.} \\ 250 & 500 & 750 \\ 400 & 800 & 1800 \end{bmatrix}}_{\substack{\mathbf{U}\,(2\times3) \\ (t)}} = \underbrace{\begin{bmatrix} \text{Paper} & 1500 \\ \text{Plastic} & 3000 \end{bmatrix}}_{\substack{\mathbf{W}\,(2\times1) \\ (\text{t})}}$	(22) and (24)
	$\underbrace{\mathbf{B}_{(4\times3)}}_{\frac{t}{\$}} = \begin{bmatrix} 1.50 & 1.83 & 1.13 \\ 3.00 & 1.67 & 2.25 \\ 1.20 & 3.33 & 1.15 \\ 2.00 & 2.67 & 3.00 \end{bmatrix}$	(18) and (25)	$\underbrace{\mathbf{V}_{(2\times3)}}_{\frac{t}{\$}} = \begin{bmatrix} 2.50 & 3.33 & 3.75 \\ 4.00 & 5.33 & 9.00 \end{bmatrix}$	(23) and (26)
(1)	$\underbrace{\mathbf{M}}_{\substack{t \\ (4\times1)}} = \begin{bmatrix} 2479.00 \\ 3749.34 \\ 3272.05 \\ 4404.20 \end{bmatrix}$	(27) *a*	$\underbrace{\mathbf{S}}_{\substack{t \\ (2\times1)}} = \begin{bmatrix} 5505.25 \\ 10737.53 \end{bmatrix}$	(27) *b*
(2)	$\underbrace{\begin{bmatrix} 2479.00 & 0 & 0 & 0 \\ 0 & 3749.34 & 0 & 0 \\ 0 & 0 & 3272.05 & 0 \\ 0 & 0 & 0 & 4404.20 \end{bmatrix}}_{\hat{\mathbf{M}}}$	(28) *a*	$\underbrace{\begin{bmatrix} 5505.25 & 0 \\ 0 & 10737.53 \end{bmatrix}}_{\hat{\mathbf{S}}}$	(28) *b*
(3)	$\underbrace{\begin{bmatrix} 0.00040 & 0 & 0 & 0 \\ 0 & 0.00027 & 0 & 0 \\ 0 & 0 & 0.00031 & 0 \\ 0 & 0 & 0 & 0.00023 \end{bmatrix}}_{\underbrace{\hat{\mathbf{M}}^{-1}}_{\frac{1}{t}}}$	(29) *a*	$\underbrace{\begin{bmatrix} 0.000182 & 0 \\ 0 & 0.000093 \end{bmatrix}}_{\underbrace{\hat{\mathbf{S}}^{-1}}_{\frac{1}{t}}}$	(29) *b*
(4)	$\underbrace{\mathbf{X}}_{\frac{1}{\$}} = \begin{bmatrix} 0.00061 & 0.00074 & 0.00045 \\ 0.00080 & 0.00044 & 0.00060 \\ 0.00037 & 0.00102 & 0.00035 \\ 0.00045 & 0.00061 & 0.00068 \end{bmatrix}$	(30) *a*	$\underbrace{\mathbf{H}}_{\frac{1}{\$}} = \begin{bmatrix} 0.00045 & 0.00061 & 0.00068 \\ 0.00037 & 0.00050 & 0.00084 \end{bmatrix}$	(30) *b*
(5)	$\mathbf{L}_{(4\times1)} = \begin{bmatrix} 0.262 \\ 0.267 \\ 0.260 \\ 0.272 \end{bmatrix}$ $\hat{\mathbf{L}}_{(4\times4)} = \begin{bmatrix} 0.262 & 0 & 0 & 0 \\ 0 & 0.267 & 0 & 0 \\ 0 & 0 & 0.260 & 0 \\ 0 & 0 & 0 & 0.272 \end{bmatrix}$	(31) *a*	$\mathbf{G}_{(2\times1)} = \begin{bmatrix} 0.272 \\ 0.279 \end{bmatrix}$ $\hat{\mathbf{G}}_{(2\times2)} = \begin{bmatrix} 0.272 & 0 \\ 0 & 0.279 \end{bmatrix}$	(31) *b*
(6)	$\hat{\mathbf{L}}^{-1}_{(4\times4)} = \begin{bmatrix} 3.814 & 0 & 0 & 0 \\ 0 & 3.749 & 0 & 0 \\ 0 & 0 & 3.849 & 0 \\ 0 & 0 & 0 & 3.670 \end{bmatrix}$	(32) *a*	$\hat{\mathbf{G}}^{-1}_{(2\times2)} = \begin{bmatrix} 3.670 & 0 \\ 0 & 3.579 \end{bmatrix}$	(32) *b*

Table 8–Numerical example with hybridism (Table 5) for throughput calculation in the retail sector (Table 6) (continued)

Step (Tab. 6)	**Input end** **Wastes contained in stocks** Supply chain (suppliers) Operation	Eq.	**Output end** **Consumption wastes** Consumption chain Operation	Eq.
(7)	$\underbrace{\mathbf{Z}_{(4\times 3)}}_{\frac{t}{\$}} = \begin{bmatrix} 5.72 & 6.99 & 4.29 \\ 11.25 & 6.25 & 8.44 \\ 4.62 & 12.83 & 4.43 \\ 7.34 & 9.79 & 11.01 \end{bmatrix} = \underbrace{\begin{bmatrix} 17.00 \\ 25.93 \\ 21.88 \\ 28.14 \end{bmatrix}}_{\mathbf{Z}_i=\sum_{j=1}^{3} z_{ij}}$	(33) *a*	$\underbrace{\mathbf{E}_{(2\times 3)}}_{\frac{t}{\$}} = \begin{bmatrix} 9.18 & 12.23 & 13.76 \\ 14.32 & 19.09 & 32.21 \end{bmatrix} = \underbrace{\begin{bmatrix} 35.17 \\ 65.62 \end{bmatrix}}_{\mathbf{E}_i=\sum_{j=1}^{3} e_{ij}}$	(33) *b*
	$\mathbf{Z}_j = \sum_{i=1}^{4} z_{ij} = [28.93 \quad 35.86 \quad 28.16]$	(38) *a*	$\mathbf{E}_j = \sum_{i=1}^{2} e_{ij} = [23.49 \quad 31.32 \quad 45.98]$	(38) *b*
(9)	$\underbrace{\mathbf{Z}^*_{(4\times 3)}}_{\frac{\$}{t}} = \begin{bmatrix} 0.17 & 0.14 & 0.23 \\ 0.09 & 0.16 & 0.12 \\ 0.22 & 0.08 & 0.23 \\ 0.14 & 0.10 & 0.09 \end{bmatrix} = \underbrace{\begin{bmatrix} 0.55 \\ 0.37 \\ 0.52 \\ 0.33 \end{bmatrix}}_{\mathbf{Z}^*_i=\sum_{j=1}^{3} z^*_{ij}}$	(35) *a*	$\underbrace{\mathbf{E}^*_{(2\times 3)}}_{\frac{\$}{t}} = \begin{bmatrix} 0.11 & 0.08 & 0.07 \\ 0.07 & 0.05 & 0.03 \end{bmatrix} = \underbrace{\begin{bmatrix} 0.26 \\ 0.15 \end{bmatrix}}_{\mathbf{E}^*_i=\sum_{j=1}^{3} e^*_{ij}}$	(35) *b*
	$\mathbf{Z}^*_j = \sum_{i=1}^{4} z^*_{ij} = [0.62 \quad 0.48 \quad 0.67]$	(39) *a*	$\mathbf{E}^*_j = \sum_{i=1}^{2} e^*_{ij} = [0.18 \quad 0.13 \quad 0.10]$	(39) *b*
	$\underbrace{[2479.00 \quad 3749.34 \quad 3272.05 \quad 4404.20]}_{\substack{\mathbf{M}'(1\times 4) \\ (t)}}$	(37) *a*	$\underbrace{[5505.25 \quad 10737.53]}_{\substack{\mathbf{S}'(1\times 2) \\ (t)}}$	(37) *b*
(10)	$\underbrace{\mathbf{C}_{(1\times 3)}}_{\$} = \begin{bmatrix} \text{Food} & \text{Hyg.} & \text{Electr.} \\ 2075.00 & 1659.55 & 2161.35 \end{bmatrix}$	(36) *a*	$\underbrace{\mathbf{K}_{(1\times 3)}}_{\$} = \begin{bmatrix} \text{Food} & \text{Hyg.} & \text{Electr.} \\ 1350.00 & 1012.50 & 733.33 \end{bmatrix}$	(36) *b*

Source: By the author.

Analysis and discussion of the results

The results displayed by matrices **Z** and **E** ((33)a-b), in t/$, offer the measure of the *throughput* in economic production. Ultimately, as defined by Daly (1974) (Eqs. (6) and (7)), this is, concerning retail, the biophysical cost of maintenance, in tonnes of residues, of the commodity stocks purchased by the suppliers (supply chain) and offered to the consumers (consumption chain). From the reciprocal of each entry in matrices **Z** and **E** ((33)*a-b*), the monetary cost, in $/t, of the throughput (matrices **Z*** and **E*** — (35)*a-b*) is finally known.

By informing the monetary cost ($) per tonne (t) of each kind of material and waste involved in the production, matrices **Z*** and **E*** ((35)*a-b*) are possibly more eloquent than their parental **Z** and **E** ((33)*a-b*). The most important, though, is that it is not possible to achieve the former without the latter. This means that the total monetary cost (matrices (36)*a-b*) imposed by wastes onto each activity sector – or, in retail, onto each consumption category – is inextricably backed by the biophysical reality of economic production.

By comparing matrices **C** and **K** ((36)*a-b*), it is easy to check which consumption category suffers the greatest monetary pressure from the natural resource base, across the supply chain (matrix **C**), and which suffers the greatest monetary impact from wastes in the consumption chain (matrix **K**). Examining Eqs. (36)*a-b* in the hypothetical example of Table 8 makes it evident that the consumption of electronics is what most impacts, in monetary terms, the supply chain (matrix **C**), while the demand for food is the most costly in monetary terms for the consumption chain (matrix **K**). However, by comparing, in Table 8, the row totals ($\mathbf{Z}_i$ and $\mathbf{E}_i$) of matrices **Z** and **E** ((33)*a-b*) with the row totals ($\mathbf{Z}_i$* and $\mathbf{E}_i$*) of matrices **Z*** and **E*** ((35)*a-b*), it can be noticed that precisely the materials and wastes with the lowest biophysical cost per unit are the ones that present the highest monetary costs per unit. This means that the increase of the ultimate efficiency (Eqs. (6) e (7)) — or the reduction of the biophysical cost per unit of economic output — involves monetary costs that can be calculated with the support of matrices **Z**, **E**, **C** and **K**.

The analysis can go deeper, as the trade-offs between many kinds of materials and wastes used by consumption category are examined (matrices **Z**, **Z***, **E**, **E*** - (33)a, (35)a, (33)b, (35)b). In the hypothetical example of Table 8, the entry z_{13} = 4.29 t/$ of matrix **Z** ((33)*a*) indicates that the lowest throughput in the supply chain results from the use of metals (i = 1) in electronic products (j = 3). Likewise, the entry e_{11} = 9.18 t/$ of matrix **E** ((33)*b*) shows that the lowest throughput in the consumption chain arises from the use of paper (i = 1) for packaging food (j = 1).

The use of less natural resources and the decrease of the unit load of wastes (matrices **Z** and **E**) indicate, according to the money-laden, prevailing economic rationality, that these materials are becoming scarce. Therefore, their price (cost) per unit increases (matrices **Z*** and **E***).

This effect is clearly observed by comparing row-matrices $\mathbf{Z}_j$ ((38)*a*) and $\mathbf{Z}_j^*$ ((39)*a*) or $\mathbf{E}_j$ ((38)*b*) and $\mathbf{E}_j^*$ ((39)*b*). The lower the use of materials (biophysical cost or throughput) per consumption category, in matrices $\mathbf{Z}_j$ and $\mathbf{E}_j$, the higher the monetary costs of these materials in the economic output of these categories, as indicated by matrices $\mathbf{Z}_j$* and $\mathbf{E}_j$*.

In fact, this mismatch between monetary and biophysical values has long been observed in economics. Inevitably, it leads to efficiency leakages, known as "rebound-effects" or "Jevons paradox" (GRAY, 1914). Such discrepancies or inefficiencies not only stimulate, via monetary prices, a predatory model of economic activity, but also demonstrate how much the monetary logic can conceal biophysical reality.

Final remarks

The essence of augmented input-output models is that they allow treating wastes and natural resources in a similar way they handle monetary transactions among economic activity sectors. The typically monetary inverse input matrix or Leontief inverse ($(I - A)^{-1}$) informs, in the standard input-output model (Table 3), by how much a given i sector's output should increase to provide the inputs required by the demand of an additional unit in user sectors (j). In short, it means that any expansion of the economy (monetary incomes and monetary value of the economic output) implies increasing demands across activity sectors.

Although well known for monetary flows, this income and output “multiplier effect” has been systematically ignored for flows of matter and energy. Just like the Leontief technical coefficients (Eq. (1) and matrix (20)) denote monetary proportions of input costs in the sector’s output, the natural technical coefficients (Eqs. (25) and (26) and matrices (18) and (23)) inform the proportions of the biophysical costs of inputs in the monetary value of the sector’s output. When these latter costs are ignored, the “multiplier effect of pollution” (Eqs. (12)*b*, (13)*b*, (27)*a-b*) is underestimated, which leads to the exhaustion of environmental sources and waste sinks, just like overlooking the multiplier effect of income and economic output determines intersectoral production bottlenecks.

The logic behind the multiplier effect of pollution is that, if the technical relationship between the economic output and the use of resources (sources of raw materials and energy) and natural services (waste sinks) are maintained, environmental pressures increase with income and economic output growth. More and more sources and natural waste sinks are required to meet the growing demand for economic commodities, stimulated by the expansion of income. Paradoxically, however, hybridism also reveals that lower biophysical impacts (costs) (matrices $\mathbf{Z}_i$ and $\mathbf{E}_i$) result in higher monetary costs (matrices $\mathbf{Z}_i$* and $\mathbf{E}_i$*) — and vice versa (Table 8).

From an environmental policy standpoint, this paradox implies that if subverting them is not an option, the goals of reducing MSW (Municipal Solid Wastes) must be defined from quantities of waste, rather than from monetary costs. On the other hand, MSW generators know better the monetary costs than the biophysical and material grounding of their economic activity.

Such an asymmetry exists because the growth of the economic output and income is split off from energy-material-saving and reducing goals. Because of the pollution multiplier effect (BUTNAR; LLOP, 2007) and of rebound effects (GRAY, 1914), the only savings that count are those in absolute (matrices **M** and **S** – Eqs. (27)*a-b*), rather than in relative terms (matrices **B** – Eqs. (18) and (25) – and **V** – Eqs. (23) and (26)). As economies mature, they tend to use less energy and materials per unit output. In other words, they become less MSW-intensive. In fact, however, this intensity decrease means an increase in the use (consumption) of energy and materials (MONIBOT, 2006), precisely because their monetary cost (matrices **Z*** and **E***) (Eqs. (35)*a-b* and (39)*a-b*) per unit input becomes *relatively* smaller, as shown by matrices **Z**, **E**, **Z*** and **E*** (Eqs. (33) *a-b*, (35)*a-b*, (38)*a-b* and (39)*a-b*).

All the same, even though the monetary logic is kept away from the biophysical foundations of reality, augmented input-output models provide a quick and ready identification of consumption categories that are most harmful to the resource base and environmental services. For sure, this type of analysis substitutes with advantages the bewildering and expensive calculations of sustainability indexes — especially as regards unmanageable devices of weighting and choosing of indicators, as required by the methodology of index-building.

Notes

i The assumption of invariability or slow change claimed by the Leontief closed model for the technical coefficients a_{ij} suggests that in nature's economy technical change (evolution) is much slower than the speed of technical progress in the human economy (Daly, 1968).

ii Macroeconomic multipliers were thoroughly studied by the British economist J. M. Keynes. Fundamentally, the income and output multiplier depends on the economy's expenditures, entailed by the marginal propensity to consumption (c). Its value is given by $1/(1 - c)$, with $0 < c < 1$.

iii This interpretation refers to the "forward linkages", in the Jones (1976) output inverse matrix. It accounts for the impacts onto users of the economic output growth resulting from an input unit increase in supplier industries (suppliers).

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"CRADLE-TO-GRAVE" SUSTAINABILITY: EXTENSION OF INPUT-OUTPUT MODELS TO URBAN SOLID WASTES AND TO CORPORATE SOCIAL AND ENVIRONMENTAL RESPONSIBILITY IN THE RETAIL SECTOR

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Resumo: Segundo uma versão mais consistente de sustentabilidade, os negócios sustentáveis definem-se não só pela responsabilidade de recolher resíduos de bens e serviços deixados ao longo da cadeia de consumo (*output end*), mas também pela de reduzi-los na cadeia de fornecedores (*input end*). Noutras palavras, trata-se de reduzir o *throughput* – o custo inevitável de manutenção dos estoques, desde a ponta dos insumos (*input*), com os requerimentos de recursos materiais e energéticos (depleção) para o suprimento de bens e serviços, até a ponta dos produtos (*output*), com os resíduos (poluição) deixados pelo caminho. Por isso as matrizes de insumo-produto permitem tratar adequadamente do *throughput*. Adicionalmente, a introdução do hibridismo nesses modelos possibilita lastrear o valor monetário (em unidades monetárias) do produto econômico ao valor biofísico (em unidades de peso ou volume) de sua manutenção. Um modelo aplicado ao setor varejista demonstra por que poupanças biofísicas implicam custos monetários maiores.

Palavras-chave: Resíduos sólidos urbanos (RSU), responsabilidade socioambiental (RSA), modelos de insumo-produto, varejo sustentável

Abstract: According to a stronger version of sustainability, sustainable business is defined by the responsibility for both collecting wastes arising from goods and services along the output end and for reducing them along the input end (supply chain). In other words, it is meant by reducing the *throughput* – the unavoidable cost of maintaining stocks, from material and energy requirements (depletion) to supply goods and services, at the input end, up to the wastes (pollution) arising from their consumption and left along the way, at the output end. Accordingly, input-output matrices can appropriately cope with the *throughput*. Moreover, by bringing hybridism into these models, it is possible to ground the monetary value (measured in monetary units) of the economic output in the biophysical value (measured in weight or volume units) of its maintenance. An alike model is applied to the retail sector to show why biophysical savings imply higher monetary costs.

Keywords: Municipal solid wastes (MSW), corporate social and environmental responsibility, input-output models, sustainable retail

Resúmen: Según una versión más consistente de sostenibilidad, los negocios sostenibles son definidos no solo por la responsabilidad de recoger desechos de bienes y servicios al largo de la cadena de consumo (*output end*), sino por la de reducirlos en la cadena de suministros (*input end*). O sea, se trata de reducir el *throughput* — el costo inevitable de manutención de stocks, desde la punta de suministros (*input*), con los requerimientos de recursos materiales y energéticos para la provisión de bienes y servicios, hacia la punta de productos (*output*), con los desechos (polución) que se quedan por el camino. Por eso, las matrices de insumo-producto permiten tratar adecuadamente del *throughput*. Además, la introducción del hibridismo en eses modelos posibilita enganchar el valor monetario (en unidades monetarias) del producto económico en el valor biofísico (en unidades de peso u volúmen) de su manutención. Un modelo aplicado al sector tendero demuestra por qué ahorros biofísicos implican costos monetarios mayores.

Palabras-clave: Desechos sólidos urbanos, responsabilidad socioambiental, modelos de insumo-produto, tiendas sostenibles