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A diferentiable manifold with (k, -(k-2)) - structure of rank r

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SUMMARY

Recently, while studying a differentiable manifold with f-structure of *rank r*, Mishra[2] has made use of a tensor as a vector valued linear function. The $\xi(K, -(K-2))$ — structure has been defined and studied by the author in[1]. The purpose of the present paper is to study a differentiable manifold with $\xi(K, -(K-2))$ — structure of *rank r*, following this approach. In this paper we have defined a metric tensor g in a differentiable manifold with $\xi(K, -(K-2))$ — structure. We have considered the cases when K is odd or even, separately. A large number of results in terms of the operators s and t have been obtained.

1. *The operators s and t*:

Let us consider an n-dimensional real differentiable manifold V_n of differentiability clase C^{r+1}. Let there exist in V_n a vector valued linear function ξ satisfying

$$(1.1) \qquad \frac{K}{X} - \frac{K-2}{X} = 0^{**}, \ (2 \text{ rank } \xi - \text{rank } \xi^{K-1}) = \dim V_n,$$

for an arbitrary vector field X, where we adopt the following notation for bar over X:

$$(1.2) \qquad \frac{K}{X} \underset{==}{def} \xi^K (X),$$

rank $(\xi) = r$ is constant everywhere and K is odd. Then ξ is called a '$\xi(K, -(K-2))$ — structure of *rank r*' and V_n is called an n-dimensional differentiable manifold with $\xi(K, -(K-2))$ — structure of *rank r*.

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** In sections 1 and 2, we have taken K odd and in sections 3 and 4, K is taken even.

Agreement (1.1). In the above and in what follows, the equations containing X, Y hold for arbitrary vector fields X, Y etc.
Let us put

$$(1.3) \qquad s(X) \underset{==}{def} \frac{K-1}{X}, \; t(X) \; def \; X - \frac{K-1}{X}$$

Now we shall prove the following:
Theorem (1.1). For a vector valued linear function ξ satisfying (1.1), the operators s and t defined by (1.3) and applied to the tangent space at each point of the manifold are complementary projection operators.
Proof. By virtue of (1.1), (1.2) and (1.3), we have

$$(1.4) \qquad s(X) + t(X) = X;$$

$$s^2(X) = s(s(X)) = \begin{matrix} xxx \\ xxx \\ xxx \end{matrix} \quad s\left(\frac{K-1}{X}\right),$$

$$= \frac{K-1}{\left(\frac{K-1}{X}\right)}{}^{***} = \frac{K-2}{\left(\frac{K}{X}\right)},$$

$$= \frac{K-2}{\left(\frac{K-2}{X}\right)} = \frac{K-4}{\left(\frac{K}{X}\right)},$$

(1.5)

$$= \frac{K-(K-1)}{\left(\frac{K-2}{X}\right)} = \frac{1}{\left(\frac{K-2}{X}\right)},$$

$$t^2(X) = t(t(X)) = t(X) - t\left(\frac{K-1}{X}\right),$$

$$= \frac{K-1}{X} = s(X);$$

*** Here and in what follows, $\frac{K\text{-}1}{\left(\frac{K\text{-}1}{X}\right)}$ means $\overset{= \dots (2K\text{-}2) \text{ times}}{X}$

$$= X - \frac{K-1}{X} - \frac{K-1}{X} - \left(\frac{\frac{K-1}{\frac{K-1}{X}}}{}\right),$$

(1.6)

$$= X - \frac{K-1}{X} = t(X);$$

$$t(s(X)) = s(t(X)) = s(X) - s\left(\frac{K-1}{X}\right),$$

$$(1.7) \qquad = \frac{K-1}{X} - \frac{\frac{K-1}{K-1}}{X} = 0.$$

This proves the theorem.

Theorem (1.2). We have

$$(1.8) \qquad s\left(\frac{K-3}{X}\right) = \frac{K-3}{s(X)} = s(X),$$

$$t\left(\frac{K-3}{X}\right) = \frac{K-3}{t(X)} = \frac{K-3}{X} - \frac{K-1}{X};$$

$$(1.9) \qquad \frac{K-2}{s(X)} = s\left(\frac{K-2}{X}\right) = \frac{K-2}{X}, \frac{K-2}{t(X)} = t\left(\frac{K-2}{X}\right) = 0;$$

$$(1.10) \qquad \text{rank}(s) = 2^{r-n}, \ \text{rank}(t) = 2n - 2r.$$

Proof. By virtue of (1.1), (1.2) and (1.3), we have

$$\frac{K-3}{s(X)} = s\left(\frac{K-3}{X}\right);$$

$$= \left(\frac{\frac{K-1}{K-3}}{X}\right) = \left(\frac{\frac{K-4}{K}}{X}\right),$$

$$(1.11) \qquad = \left(\frac{\frac{K-4}{K-2}}{X}\right) = \left(\frac{\frac{K-6}{K}}{X}\right),$$

$$= \left(\frac{\frac{K-(K-1)}{K-2}}{X}\right) = \left(\frac{\frac{1}{K-2}}{X}\right),$$

$$= \quad \frac{K-1}{X} \quad = s(X);$$

(1.12) $$\frac{K-3}{t(X)} = t\left(\frac{K-3}{X}\right) = \frac{K-3}{X} - \left(\frac{\frac{K-1}{K-3}}{X}\right) = \frac{K-3}{X} - \frac{K-1}{X}.$$

Barring X in (1.11) and using (1.1) and (1.3), we obtain

$$\frac{K-2}{s(X)} = s\left(\frac{K-2}{X}\right) = \frac{K}{X} = \frac{K-2}{X}.$$

Barring X in (1.12) and using (1.1) and (1.3), we obtain

$$\frac{K-2}{t(X)} = t\left(\frac{K-2}{X}\right) = \frac{K-2}{X} - \frac{K}{X} = 0.$$

The proof of (1.10) follows directly by virtues of the equations (1.1), (1.2) and (1.3).

Let π_{2r-n} and π_{2n-2r} be the complementary distributions corresponding to the projection operators s and t respectively. Then π_{2r-n} and π_{2n-2r} are $(2r-n)$ and $(2n-2r)$-dimensional. Obviously, $n \leq 2r \leq 2n$.

Remark (1.1). If rank $(\xi) = n$, then from (1.10) $t = 0$. In this case (1.3) reduces to

(1.13) $$X - \frac{K-1}{X} = 0.$$

Barring X twice in (1.13), we have

$$\frac{2}{X} - \left(\frac{\frac{1}{K}}{X}\right) = 0$$

or,

$$\frac{2}{X} - \frac{K-1}{X} = 0,$$

in consequence of (1.1). This in view of (1.13) yields

(1.14) $$\frac{2}{X} - X = 0.$$

From which we conclude that ξ is an almost product structure.

2. $\xi(K, -(K-2))$*-structure*:

In this section, we shall study some results connected with the $\xi(K,-(K-2))$-structure when K is odd.| We shall also define a metric tensor g in a differentiable manifold with $\leftrightarrow$ $\xi(K, -(K-2))$-structure.

Theorem (2.1). $\xi(K,-(K-2))$-structure is not unique. Let τ be a non-singular vector valued linear function in V_n. Then η defined by

$$\tau(\eta(X)) \underset{=}{\text{def}} \frac{1}{\tau(X)} \tag{2.1}$$

is also $\xi(K,-(K-2))$-structure.

Proof. In consequence of (2.1), we have

$$\frac{K-1}{\tau(\eta(X))} = \frac{K-2}{\tau(\eta^2(X))} = \frac{K-3}{\tau(\eta^3(X))},$$

(2.2)

$$= \frac{K-K}{\tau(\eta^K(X))} = \tau(\eta^K(X)).$$

Also from (1.1) and (2.1), we have

$$\frac{K-1}{\tau(\eta(X))} = \frac{K}{\tau(X)} = \frac{K-2}{\tau(X)},$$

$$= \frac{K-K}{\tau(\eta(X))} = \frac{K-4}{\tau(\eta^2(X))}, \tag{2.3}$$

$$= \frac{K-K}{\tau(\eta^{K-2}(X))} = \tau(\eta^{K-2}(X))$$

From (2.2) and (2.3), we have

$$\tau(\eta^K(X) - \eta^{K-2}(X)) = 0.$$

Since τ is non-singular, we obtain

$$\eta^{K}(X) - \eta^{K-2}(X) = 0.$$

This proves the statement.

Theorem (2.2). Let S and T be the operators for η corresponding to the operators s and t for $\xi(K, -(K-2))$-structure respectively. Then we have

$$\tau(S(X)) = s(\tau(X)),\ \tau(T(X)) = t(\tau(X)); \tag{2.4}$$

$$\tau(S(X) + T(X)) = \tau(X). \tag{2.5}$$

Proof. By virtue of (1.2), (1.3) and (2.1), we have

$$\tau(S(X)) = \tau(\eta^{K-1}(X)) = \frac{1}{\tau(\eta^{K-2}(X))},$$

$$= \frac{2}{\tau(\eta^{K-3}(X))} = \frac{3}{\tau(\eta^{K-4}(X))},$$

$$= \frac{K-1}{\tau(\eta^{K-K}(X))} = \frac{K-1}{\tau(X)},$$

$$= s(\tau(X));$$

$$\tau(T(X)) = \tau(X) - \tau(\eta^{K-1}(X)) = \tau(X) - \frac{K-1}{\tau(X)} = t(\tau(X)).$$

The proof of (2.5) follows directly by virtue of the equations (1.4) and (2.4).

Theorem (2.3). We have

$$\tau(S^2(X)) = s(\tau(X)),\ \tau(T^2(X)) = t(\tau(X)); \tag{2.6}$$

$$\tau(S(T(X))) = \tau(T(S(X))) = 0. \tag{2.7}$$

Proof. The proof of the theorem follows by virtue of the equations (1.5), (1.6), (1.7) and (2.4).

Theorem (2.4). We have

$$(2.8)\qquad \overset{K-3}{\overline{\tau(S(X))}} = s(\tau(X)),\ \overset{K-3}{\overline{\tau(T(X))}} = \overset{K-3}{\overline{t(\tau(X))}};$$

$$(2.9)\qquad \overset{K-2}{\overline{\tau(S(X))}} = \overset{K-2}{\overline{s(\tau(X))}},\ \overset{K-2}{\overline{\tau(T(X))}} = 0.$$

Proof. The proof of the theorem follows by virtue of the equations (1.8), (1.9), and (2.4).

In the manifold V_n with $\xi(K,—(K—2))$ — structure, we can always introduce a metric tensor g as follows. Let

$$(2.10)\quad g(\overset{K-2}{\overline{X}}, \overset{K-2}{\overline{Y}}) \underset{=}{\mathrm{def}}\ g(\overset{K-1}{\overline{X}}, \overset{K-3}{\overline{Y}}) \underset{=}{\mathrm{def}}\ (\overset{K-3}{\overline{X}}, \overset{K-1}{\overline{Y}}).$$

Since

(i) g is symmetric and
(ii) repeated operation of barring X or Y in (2.10) yields the same set of equations and there is no contradiction; therefore, we are justified in assuming g as given in (2.10).

Let es put

$$(2.11)\ t^*(\overset{K-3}{\overline{X}}, \overset{K-3}{\overline{Y}}) \underset{=}{\mathrm{def}}\ g(t(\overset{K-3}{\overline{X}}), \overset{K-3}{\overline{Y}}) \underset{=}{\mathrm{def}}\ g(\overset{K-3}{\overline{X}}, t(\overset{K-3}{\overline{Y}})).$$

Then by virtue of 1.8 and (2.10), equation (2.11) becomes

$$(2.12)\qquad t^*(\overset{K-3}{\overline{X}}, \overset{K-3}{\overline{Y}}) = g(\overset{K-3}{\overline{X}}, \overset{K-3}{\overline{Y}}) - g(\overset{K-2}{\overline{X}}, \overset{K-2}{\overline{Y}}).$$

Therefore

$$(2.13)\quad g(\overset{K-2}{\overline{X}}, \overset{K-2}{\overline{Y}}) = g(\overset{K-3}{\overline{X}}, \overset{K-3}{\overline{Y}}) - t^*(\overset{K-3}{\overline{X}}, \overset{K-3}{\overline{Y}}).$$

Equations (2.10) and (2.13), in consequence of (1.1), (1.9) and (2.11), are both equivalent to

$$(2.14)\qquad g(\overset{K-1}{\overline{X}}, \overset{K-1}{\overline{Y}}) = g(\overset{K-2}{\overline{X}}, \overset{K-2}{\overline{Y}}).$$

From (2.10) we also have

$$g(\overset{K-1}{\overline{X}}, \overset{K-2}{\overline{Y}}) - g(\overset{K-2}{\overline{X}}, \overset{K-1}{\overline{Y}}) = 0. \tag{2.15}$$

In consequence of (1.1) and (2.10), equation (2.15) becomes

$$g(\overset{K-2}{\overline{X}}, \overset{K-3}{\overline{Y}}) - g(\overset{K-3}{\overline{X}}, \overset{K-2}{\overline{Y}}) = 0. \tag{2.16}$$

Theorem (2.5). Let G be the metric for η corresponding to the metric g for $\xi\,(K, -(K-2)$ — structure, such that

$$G\,(X, Y) \underset{=}{\text{def}}\, g(\tau\,(X), \tau\,(Y)). \tag{2.17}$$

Then G also satisfies an equation of the form (2.13), i.e.

$$\begin{aligned} G\,(\eta^{K-2}(X), \eta^{K-2}(Y)) &= \\ G\,(\eta^{K-3}(X), \eta^{K-3}(Y)) &- \\ - T^{*}(\eta^{K-3}(X), \eta^{K-3}(Y)), \end{aligned} \tag{2.18}$$

where

$$\begin{aligned} &T^{*}\,(\eta^{K-3}(X), \eta^{K-3}(Y)), \\ &\underset{=}{\text{def}}\, G(T(\eta^{K-3}(X)), \eta^{K-3}(Y)), \\ &\underset{=}{\text{def}}\, G(\eta^{K-3}(X), T(\eta^{K-3}(Y))). \end{aligned} \tag{2.19}$$

Proof. By virtue of (2.3), (2.8), (2.11), 2.13) and (2.17), we have

$$\begin{aligned} &G\,(\eta^{K-2}(X), \eta^{K-2}(Y)) \\ &= g\,(\tau\,(\eta^{K-2}(X)), \tau\,(\eta^{K-2}(Y))), \\ &= g(\overset{K-2}{\overline{\tau(X)}}, \overset{K-2}{\overline{\tau(Y)}}), \\ &= g(\overset{K-3}{\overline{\tau(X)}}, \overset{K-3}{\overline{\tau(Y)}}) - \\ &- t^{*}(\overset{K-3}{\overline{\tau(X)}}, \overset{K-3}{\overline{\tau(Y)}}), \end{aligned} \tag{2.20}$$

$$= g(\overset{K-3}{\overline{\tau(X)}}, \overset{K-3}{\overline{\tau(Y)}}) —$$

$$— g(t(\overset{K-3}{\overline{\tau(X)}}), \overset{K-3}{\overline{\tau(Y)}}),$$

$$= g(\overset{K-3}{\overline{\tau(X)}}, \overset{K-3}{\overline{\tau(Y)}}) —$$

$$— g(\overset{K-3}{\overline{\tau(T(X))}}, \overset{K-3}{\overline{\tau(Y)}}).$$

Now from (2.1) we have

$$\overset{K-3}{\overline{\tau(X)}} = \overset{K-4}{\overline{\tau(\eta(X))}} =$$

$$= \overset{K-5}{\overline{\tau(\eta^2(X))}},$$

(2.21)

$$= \overset{K-K}{\overline{\tau(\eta^{K-3}(X))}} = \tau(\eta^{K-3}(X)).$$

Therefore, in consequence of (2.17), (2.19) and (2.21), equation (2.20) becomes

$$G(\eta^{K-2}(X), \eta^{K-2}(Y))$$

$$= g(\tau(\eta^{K-3}(X)), \tau(\eta^{K-3}(Y))) —$$

$$— g(\tau(\eta^{K-3}(T(X))), \tau(\eta^{K-3}(Y))),$$

$$= G(\eta^{K-3}(X), \eta^{K-3}(Y)) —$$

$$— G(T(\eta^{K-3}(X)), \eta^{K-3}(Y)),$$

$$= G(\eta^{K-3}(X), \eta^{K-3}(Y)) —$$

$$— T^*(\eta^{K-3}(X)), \eta^{K-3}(Y)).$$

Thus G satisfies an equation of the form (2.13).

3. *The operators s and t (K even)*:

Let us consider an n-dimensional real differentiable manifold V_n of differentiability class C^{r+1}. Let there exist in V_n a vector valued linear function ξ satisfying

$$(3.1) \qquad \overset{K}{\overline{X}} - \overset{K-2}{\overline{X}} = 0, \; (2 \operatorname{rank} \xi - \operatorname{rank} \xi^{K-2}) = \dim V_n,$$

for an arbitrary vector field X, where we adopt the following notation for bar over X:

$$(3.2) \qquad \overset{K}{\overline{X}} \underset{=}{\text{def}}\, \xi^K(X),$$

rank $(\xi) = r$ is constant everywhere and K is even. Then ξ is called a 'ξ $(K, -$ $-(K-2))$ — structure of rank r' and V_n is called an n-dimensional differentiable manifold with ξ $(K, -(K-2))$ — structure of rank r.

Agreement (3.1). In the above and in what follows, the equations containing X, Y hold for arbitrary vector fields X, Y, etc.

Let us put

$$(3.3) \qquad s(X) \underset{=}{\text{def}} \overset{K-2}{\overline{X}}, \; t(X) \underset{=}{\text{def}} X - \overset{K-2}{\overline{X}}.$$

Thus we have

Theorem (3.1). For a vector valued linear function ξ satisfying (3.1), the operators s and t defined by (3.3) and applied to the tangent space at each point of the manifold are complementary projection operators.

Theorem (3.2). We have

$$(3.4) \qquad \begin{cases} s(\overset{K-3}{\overline{X}}) = (\overset{K-3}{\overline{s(X)}}) = \overset{K-1}{\overline{X}}, \\ t(\overset{K-3}{\overline{X}}) = \overset{K-3}{\overline{t(X)}} = \overset{K-3}{\overline{X}} - \overset{K-1}{\overline{X}}; \end{cases}$$

$$(3.5) \qquad \overset{K-2}{\overline{s(X)}} = \overset{K-2}{\overline{s(X)}} = s(X), \; \overset{K-2}{\overline{t(X)}} = \overset{K-2}{\overline{t(X)}} = 0;$$

$$(3.6) \qquad \operatorname{rank}(s) = 2r\text{-}n, \; \operatorname{rank}(t) = 2n\text{-}2r.$$

The proofs of theorems (3.1) and (3.2) follow from the pattern of the proofs of theorems (1.1) and (1.2) respectively.

Let π_{2r-n} and π_{2n-2r} be the complementary distributions corresponding to the projection operators s and t respectively. Then π_{2r-n} and π_{2n-2r} are $(2r\text{-}n)$ and $(2n - 2r)$ — dimensional. Obviously, $n \leq 2r \leq 2n$.

Remark (3.1). If rank $(\xi) = n$, then from (3.6) $t = 0$. In this case (3.3) reduces to

$$X = \frac{K-2}{X} = 0. \tag{3.7}$$

Barring X twice in (3.7), we have

$$\frac{2}{X} = \frac{K}{X} = 0$$

or,

$$\frac{2}{X} = \frac{K-2}{X} = 0,$$

in consequence of (3.1). This in view of (3.7) yields

$$\frac{2}{X} - X = 0. \tag{3.8}$$

From which we conclude that ξ is an almost product structure.

4. $\xi\,(K, -(K-2))$ — *structure (K even):*

In this section, we shall study some results connected with the ξ $(K, -$ $-(K-2))$ — structure when K is even. We shall also define a metric tensor g in a differentiable manifold with ξ $(K, -(K-2))$ — structure.

Now we have the following theorems:

Theorem (4.1). $\xi\,(K, -(K-2))$ — structure is not unique. Let τ be a non-singular vector valued linear function in V_n. Then η defined by
is also $\xi\,(K, -(K-2))$ — structure.

$$\tau\,(\eta\,(X)) \underset{=}{\text{def}} \frac{1}{\tau(X)} \tag{4.1}$$

Theorem (4.2). Let S and T be the operators for η corresponding to the operators s and t for $\xi\,(K, -(K-2))$ — structure $\longleftrightarrow$ respectively. Then we have

$$\tau\,(S(X)) = s(\tau(X)), \tau(T(X)) = t(\tau(X))\,; \tag{4.2}$$

$$\tau\,(S(X) + T\,(X)) = \tau\,(X). \tag{4.3}$$

Theorem (4.3). We have

$$\tau\,(S^2(X)) = s\,(\tau(X)), \tau\,(T^2(X)) = t\,(\tau(X))\,; \tag{4.4}$$

$$\tau\,(S(T(X))) = \tau\,(T(S(X))) = 0. \tag{4.5}$$

Theorem (4.4). We have

$$\overset{K-3}{\overline{\tau\,(S(X))}} = \overset{K-3}{\overline{s\,(\tau(X))}}, \overset{K-3}{\overline{\tau\,(T(X))}} = \overset{K-3}{\overline{t\,(\tau(X))}}\,; \tag{4.6}$$

$$\overset{K-2}{\overline{\tau\,(S(X))}} = s\,(\tau(X)), \overset{K-2}{\overline{\tau\,(T(X))}} = 0. \tag{4.7}$$

The proofs of theorems (4.1), (4.2), (4.3) and (4.4) are similar to those of theorems (2.1), (2.2), (2.3) and (2.4) respectively.

In the manifold V_n with $\xi\,(K, -(K-2))$ — structure, we can always introduce a metric tensor g as follows. Let

$$g(\overset{K-2}{\overline{X}}, \overset{K-2}{\overline{Y}}) \overset{\text{def}}{=} g(\overset{K-1}{\overline{X}}, \overset{K-3}{\overline{Y}}) \overset{\text{def}}{=} g(\overset{K-3}{\overline{X}}, \overset{K-1}{\overline{Y}}). \tag{4.8}$$

Since

(i) g is symmetric and

(ii) repeated operation of barring X or Y in (4.8) yields the same set of equations and there is no contradiction; therefore, we are justified in assuming g as given in (4.8).

Let us put

$$t^*(\overset{K-3}{\overline{X}}, \overset{K-3}{\overline{Y}}) \overset{\text{def}}{=} \tag{4.9}$$

$$g\,(t\,(\overset{K-3}{\overline{X}}), \overset{K-3}{\overline{Y}}) \overset{\text{def}}{=} g\,(\overset{K-3}{\overline{X}}, t\,(\overset{K-3}{\overline{Y}})).$$

Then by virtue of (3.4) and (4.8), equation (4.9) becomes

$$(4.10)\qquad t^*(\overset{K-3}{\overline{X}}, \overset{K-3}{\overline{Y}}) = g(\overset{K-3}{\overline{X}}, \overset{K-3}{\overline{Y}}) - g(\overset{K-2}{\overline{X}}, \overset{K-2}{\overline{Y}})$$

Therefore

$$(4.11)\qquad g(\overset{K-2}{\overline{X}}, \overset{K-2}{\overline{Y}}) = g(\overset{K-3}{\overline{X}}, \overset{K-3}{\overline{Y}}) - t^*(\overset{K-3}{\overline{X}}, \overset{K-3}{\overline{Y}})$$

Equations (4.8) and (4.11), in consequence of (3.1), (3.5) and (4.9), are both equivalent to

$$(4.12)\qquad g(\overset{K-1}{\overline{X}}, \overset{K-1}{\overline{Y}}) = g(\overset{K-2}{\overline{X}}, \overset{K-2}{\overline{Y}})$$

From (4.8) we also have

$$(4.13)\qquad g(\overset{K-1}{\overline{X}}, \overset{K-2}{\overline{Y}}) - g(\overset{K-2}{\overline{X}}, \overset{K-1}{\overline{Y}}) = 0$$

In consequence of (3.1)and (4.8), equation (4.13) becomes

$$(4.14)\qquad g(\overset{K-2}{\overline{X}}, \overset{K-3}{\overline{Y}}) - g(\overset{K-3}{\overline{X}}, \overset{K-2}{\overline{Y}}) = 0$$

Theorem (4.5). Let G *be the metric for* η *corresponding to the metric* g *for* $\xi(K,-(K-2))$ —structure, such that

$$(4.15)\qquad G(X,Y) \underset{=}{\text{def}} g(\tau(X), \tau(Y)).$$

Then G also satisfies an equation of the form (4.11), i.e.

$$(4.16) \qquad G(\eta^{K-2}(X), \eta^{K-2}(Y)) = G\ \eta^{K-3}(X), \eta^{K-3}(Y) \ - - T^{*}(\eta^{K-3}(X), \eta^{K-3}(Y)),$$

where

$$(4.17) \qquad T^{*}(\eta^{K-3}(X), \eta^{K-3}(Y)) \underset{=}{\text{def}} G(T(\eta^{K-3}(X)), \eta^{K-3}(Y)), \underset{=}{\text{def}} G(\eta^{K-3}(X), T(\eta^{K-3}(Y))).$$

Proof. The proof of the theorem follows from the pattern of the proof of theorem (2.5).

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