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Some integrals involving generalized H-function of Fox*

by

T.N. Srivastava**

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1. INTRODUCTION

The H-function as introduced by Fox [6] has been generalized and studied by many mathematicians notably Munot and Kalla [8], Gupta [4] and Saxena [11]. The generalized H-function of two variables includes, as special cases, a single H-function, product of two H-functions of single variable and most of the known functions of one or two variables. (eg.Meijer's G-function and Macrobert's E-function, etc.) This function plays an important role ni Mathematics and can be applied in Physics and Statistics. The object of this paper is to evaluate certain finite integrals involving the product of H-function of two variables and the hypergeometric function. An attempt has been made to unify certain results in the theory of Agarwal's G-function of two variables [1]. Some particular cases have also been pointed out.

2. DEFINITION OF A GENERALIZED H-FUNCTION

In the notation of Saxena [11] the generalized H-function of two variables can be defined in terms of double Mellin-Barnes integral as follows

$$(2.1)\qquad H\left[\begin{matrix}x\\ y\end{matrix}\right] = H^{l,(n,q),*,(m,p)}_{E,(A,C),F,(B,D)}\left[\begin{matrix}x\\ \\ y\end{matrix}\left|\begin{matrix}[e:\rho,\ \varnothing]\\ [a,\alpha]:[c,\gamma]\\ [f:\sigma,\Psi]\\ [b,\beta],[d,\delta]\end{matrix}\right.\right]$$

$$= \frac{1}{(2\pi i)^2}\int_{-i\infty}^{i\infty}\int_{-i\infty}^{i\infty} X_1(\xi)X_2(\eta)X_3(\xi,\eta)\,x^{-\xi}y^{-\eta}\,d\xi d\eta$$

** Department of Mathematics, Concordia University Loyola Campus, Montreal, Canada.

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where

$$X_1(\xi) = \frac{\prod_{j=1}^{m} \Gamma(b_1 + \xi\beta_1) \prod_{j=1}^{n} \Gamma(1 - a_j - \xi\alpha_j)}{\prod_{j=m+1}^{B} \Gamma(1 - b_j - \xi\beta_j) \prod_{j=n+1}^{A} \Gamma(a_j + \xi\alpha_j)}$$

$$X_2(\eta) = \frac{\prod_{j=1}^{p} \Gamma(d_j + \eta\delta_j) \prod_{j=1}^{q} \Gamma(1 - c_j - \eta\gamma_j)}{\prod_{j=p+1}^{D} \Gamma(1 - d_j - \eta\delta_j) \prod_{j=q+1}^{C} \Gamma(C_j + \eta\gamma_j)}$$

$$X_3(\xi,\eta) = \frac{\prod_{j=1}^{l} \Gamma(e_j - \xi\rho_j - \eta\varnothing_j)}{\prod_{j=l+1}^{E} \Gamma(1 - e_j + \xi\rho_j + \eta\varnothing_j) \prod_{j=1}^{F} \Gamma(f_j - \xi\sigma_j - \eta\Psi_j)}$$

and the empty product is interpreted as unity. The following assumptions are made

(i) $0 \leq n \leq A,\ 1 \leq m \leq B,\ 0 \leq q \leq C,\ 1 \leq p \leq D,\ 0 \leq l \leq E$

(ii) l,m,n,p,q and A,B,C,D,E,F are non negative integers.

(iii) The parameters a's, b's, c's, d's, e's and f's are complex numbers where as

α's, β's, γ's, δ's, $\varnothing$'s and Ψ's are real and positive.

(iv) All the poles of the integrand in (2.1) are simple.

(v) The sequence of parameters (a_A), (α_A), (b_B), (β_B); are such that none of the poles of the integrand coincide.

(vi) The path of integration is intended, if necessary, in such a manner that all the poles of $\Gamma(b_j + \xi\beta_j)$ for $j = 1, \ldots m$, $\Gamma(d_j + \eta\delta_j)$ for $j = 1 \ldots p$ and $\Gamma(e_j - \xi\rho_j - \eta\varnothing_j)$ for $j = 1 \ldots l$, lie to the left and those of $\Gamma(1 - a_j - \xi\alpha_j)$ for $j = 1 \ldots m$ and $\Gamma(1 - c_j - \eta\gamma_j)$ for $j = 1 \ldots q$ lie to the right of the imaginary axis.

(vii) The integral converges if

(2.2)
$$\sum^{E} \varnothing_j + \sum^{B} \beta_j - \sum^{k} \Psi_j - \sum^{A} \alpha_j \leq 0$$

$$\sum^{E} \varnothing_j + \sum^{D} \delta_j - \sum^{F} \Psi_j - \sum^{C} \gamma_j \leq 0$$

$$|\arg x| < \frac{\pi}{2}\lambda_1 \quad \text{and} \quad |\arg y| < \frac{\pi}{2}\lambda_2$$

where

(2.3)
$$\lambda_1 = \sum_{1}^{m} \beta_j - \sum_{m+1}^{B} \beta_j + \sum_{1}^{n} \alpha_j - \sum_{n+1}^{A} a_j + \sum_{1}^{l} \varnothing_j - \sum_{l+1}^{E} \varnothing_j - \sum_{1}^{F} \Psi_j > 0$$

$$\lambda_2 = \sum_{j=1}^{p} \delta_j - \sum_{p+1}^{D} \delta_j + \sum_{1}^{q} \gamma_j - \sum_{q+1}^{C} \gamma_j + \sum_{1}^{l} \varnothing_j - \sum_{l+1}^{E} \varnothing_j - \sum_{1}^{F} \Psi_j > 0$$

Here as well as in what follows it is supposed that there are A of the parameters A and B of the parameters b and β etc. Thus $[a, \alpha]$ is taken to take to denote the sequence of A pairs of parameters

$$(a_1, \alpha_1), (a_2, \alpha_2), \ldots (a_A, \alpha_A)$$

and similar interpretations for $[b, \beta]$, $[e : \rho, \varnothing]$ etc. For the sake of brevity the pair of parameters like $\alpha + \beta$, α-β will be written as $\alpha \pm \beta$ and the symbol $\Delta(k, a)$ is taken to denote the set of k-parameters

$$\frac{a}{k}; \frac{a+1}{k}, \ldots, \frac{a+k-1}{k}.$$

We recall here that origin is the singularity of $H\left[{x \atop y}\right]$. The behaviour of $H\left[{x \atop y}\right]$ in the neighbourhood of $x = y = 0$ is given by

$$H\left[{x \atop y}\right] = 0\, |x|^{\mu}\, |y|^{\nu}$$

where $\mu = min\, (\frac{b_i}{\beta_i})$ and $\nu = min\, (\frac{d_j}{d_j})$ for $i = 1, \ldots, m, j = 1 \ldots$ p.

Special cases of (2.1) are given below.

$$H^{0,(n,q),*,(m,p)}_{0,(A,C),0,(B,D)}\left[\begin{matrix}x\\ \\ y\end{matrix}\middle|\begin{matrix}\cdots\\ [a,\alpha],[c,\gamma]\\ -\\ [b,\beta],[d,\delta]\end{matrix}\right]$$

(2.4)

$$\equiv H^{m,n}_{A,B}\left[x\middle|\begin{matrix}[a,\alpha]\\ [b,\beta]\end{matrix}\right] \quad H^{p,q}_{C,D}\left[y\middle|\begin{matrix}[c,\gamma]\\ [d,\delta]\end{matrix}\right]$$

$$H^{l,(n,q),*,(m,p)}_{E,(A,C),F,(B,D)}\left[\begin{matrix}x\\ \\ \\ y\end{matrix}\middle|\begin{matrix}(e:1,1)\\ (a,1)\ (c,1)\\ (f:1,1)\\ (b,1)\ (d,1)\end{matrix}\right]$$

(2.5)

$$\equiv G^{l,(n,q),*,(m,p)}_{E,(A,C),F,(B,D)}\left[\begin{matrix}x\\ \\ y\\ \end{matrix}\middle|\begin{matrix}(e)\\ (a)\ (c)\\ (f)\\ (b)\ (d)\end{matrix}\right]$$

In equation (2.4) $H^{m,n}_{p,q}\left[x\middle|\begin{matrix}[a,\alpha]\\ [b,\beta]\end{matrix}\right]$ denotes the H-function of Fox [5] and in (2.5) the right hand side is the well known Agarwal's G-function of two variables and may be denoted by $G\left[\begin{smallmatrix}x\\ y\end{smallmatrix}\right]$.

3. FINITE INTEGRALS:

(3.1)
$$\int_0^t x^{\mu-1}(t-x)^{\nu-1}\,{}_rF_s\left[\begin{matrix}(u_r)\\ (v_s)\end{matrix}\middle|Px^{\theta}\right] H\left[\begin{matrix}yx^{\lambda_1}\ (t-x)^{\omega_1}\\ zx^{\lambda_2}\ (t-x)^{\omega_2}\end{matrix}\right]dx$$

$$\equiv t^{\mu+\nu-1}\sum_{j=0}^{\infty}\frac{(u_1)_j\ldots(u_s)_j\,(Pt^{\theta})^j}{(v_1)_j\ldots(v_s)_j\quad j!}$$

$$* H^{l+2,(n,q),*,(m,p)}_{E+2,(A,C),F+1,(B+D)}\left[\begin{matrix}y\,t^{\lambda_1+\omega_1}\\ \\ \\ z\,t^{\lambda_2+\omega_2}\\ \\ \end{matrix}\middle|\begin{matrix}[e:p,\varnothing],(g_1\,\omega_1,\omega_2),(g_2:\lambda_1,\lambda_2)\\ \\ [e,\alpha];[c,\gamma]\\ [f,\sigma,\Psi],(h:\lambda_1+\omega_1,\lambda_2+\omega_2)\\ \\ [b,\beta];[d,\delta]\end{matrix}\right]$$

where $g_1 = \nu, g_2 = \mu + j\theta, r \leq s, R(\mu) > 0, R(\nu) > 0$

$$R[\mu + \lambda_1 \min(\frac{b_i}{\beta_i}) + \lambda_2 \min(\frac{d_i'}{\delta_i'})]$$

$$> 0 \; for \; i = 1, \ldots, m, i = 1, \ldots, p$$

$$R\{\nu + \omega_1 \min(\frac{b_i}{\beta_i}) + \omega_2 \min(\frac{d_i'}{\delta_i'})]$$

$$> 0 \; for \; i = 1, \ldots, m, i = 1, \ldots, p$$

and (2.2.) holds

$$(3.2) \qquad \int_0^t x^{\mu-1} (t-x)^{\nu-1} {}_rF_s \left[\begin{matrix}(u_r)\\(v_s)\end{matrix} \middle| Px^{\Theta_1}(t-x)^{\Theta_2}\right] H \left[\begin{matrix} y x^{\lambda_1}(t-x)^{\omega_1} \\ z x^{\lambda_2}(t-x)^{\omega_2}\end{matrix}\right]$$

$$= t^{\mu+\nu-1} \sum_{j=0}^{\infty} \frac{(u_1)_j \ldots (u_r)_j}{(v_1)_j \ldots (v_s)_j} \quad \frac{p^j t^{(\Theta_1+\Theta_2)j}}{j!}$$

$$\times H^{l+2,(n,q),*,(m,p)}_{E+2,(A,C),F+1,(B,'D)} \left[\begin{matrix} y t^{\lambda_1+\omega_1} \\ \\ z t^{\lambda_2+\omega_2} \end{matrix} \middle| \begin{matrix} [e:p,\varnothing],(g_1;\lambda_1,\lambda_2),(g_2;'\omega_1,\omega_2) \\ [a,\alpha);[c,\gamma] \\ [f:\sigma,\Psi],(h;\lambda_1+'\omega_1,\lambda_2+\omega_2 \\ [b;\beta];[d,\delta] \end{matrix}\right]$$

where $g_1 = \mu + j\theta_1, g_2 = \nu + j\theta_2, h = \mu + \nu + j(\theta_1 + \theta_2) \; r \leq s, R(\mu) > 0, R(\nu) > 0$

$$R\{\mu + \lambda_1 \min \frac{b_i}{\beta_i} + \lambda_2 \min \frac{d_i'}{\delta_i'}] \quad > 0 \; for \; i = 1, \ldots, m; i' = 1, \ldots p.$$

$$R[\nu + \omega_1 \min \frac{b_i}{\beta_i} + \omega_2 \min \frac{d_i'}{\delta_i'}] \quad > 0 \; for \; i = 1, \ldots, m; i' = 1, \ldots, p.$$

and (2.2) holds. θ_1 and θ_2 being positive integers (either θ_1 are θ_2 may be zero).

$$(3.3) \quad \int_0^t t^{\mu-1} (1-t)^{\mu-v} {}_2F_1 \left[\begin{matrix} u, 1-u \\ v \end{matrix} \Big| \right] H \left[\begin{matrix} x t^{\lambda_1} (1-t)^{\lambda_1} \\ y t^{\lambda_2} (1-t)^{\lambda_2} \end{matrix} \right] dx.$$

$$= \frac{\pi\, 2^{1-2\mu} (v)}{\Gamma(\frac{u+v}{2})\, \Gamma(\frac{1-u-v}{2})}$$

$$\times H^{l+2, (n,q), *, (m,p)}_{E+2, [A,C], F+2, (B,D)} \left[\begin{matrix} \frac{x}{2^{\lambda_1}} \\ \frac{y}{2^{\lambda_2}} \end{matrix} \middle| \begin{matrix} [e:\rho,\varnothing], (g_1:\lambda_1,\lambda_2)\ (g_2:\lambda_1,\lambda_2) \\ [a,\alpha]:[c,\gamma] \\ [f:\sigma,\Psi], (h_1:\lambda_1,\lambda_2), (h_2:\lambda_1,\lambda_2) \\ [b,\beta]:[d,\delta] \end{matrix} \right]$$

where $g_1 = \mu, g_2 = v+1, h_1 = \mu + \dfrac{u-v+1}{2}, h_2 = \mu - \dfrac{u+v}{2}, R(\mu) > 0,$

$R[(\mu - v + 1) > 0, R[\mu - v + \lambda_1 \min \dfrac{b_i}{\beta_i} + \lambda_2 \min \dfrac{d_j}{\delta_j}] > 0$ for $i = 1, \ldots, m, j = 1, \ldots p$

$R\{\mu + \lambda_1 \min \dfrac{b_i}{\beta_i} + \lambda_2 \min \dfrac{d_i'}{\delta_i'}] > 0$ for $i = 1, \ldots,$ and (2.2) holds.

$$(3.4) \quad \int_0^t t^{\mu-1}(1-t)^{\mu-1}\ {}_2F_1 \left[\begin{matrix} u,v \\ \frac{u+v+1}{2} \end{matrix} \middle| t \right] H \left[\begin{matrix} x t^{\lambda_1} (t-x)^{\lambda_1} \\ y t^{\lambda_2} (t-y)^{\lambda_2} \end{matrix} \right] dt$$

$$= \frac{\pi}{{}_2 2\mu - 1} \frac{\Gamma \frac{u+v+1}{2}}{\Gamma \frac{u+1}{2}\ \Gamma \frac{v+1}{2}}$$

$$* H^{l+2, (n,q), *, (m,p)}_{E+2, (A,C), F+2, (B,D)} \left[\begin{matrix} \frac{x}{2\lambda_1} \\ 2 \\ \frac{y}{2\lambda_2} \\ 2 \end{matrix} \middle| \begin{matrix} [e:\rho,\varnothing], (g_1:\lambda_1,\lambda_2), (g_2:\lambda_1,\lambda_2) \\ [a,\alpha]:[c,\gamma] \\ [f,\sigma,\Psi], (h_1:\lambda_1\,\lambda_2), (h_2:\lambda_1,\lambda_2) \\ [b,\beta]:[d,\delta] \end{matrix} \right]$$

where $g_1 = \mu, g_2 = \dfrac{1-u-v}{2} + \mu, h_1 = \dfrac{1-u}{2} + \mu, h_2 = \dfrac{1-v}{2} + \mu, R(u) > 0$

$R\,(u + v + 1) > 0, R[\mu + \lambda_1 \min \dfrac{b_i}{\beta_i} + \lambda_2 \min \dfrac{d_j}{\delta_j}] > 0$ for $i = 1, \ldots, m, j =$ $= 1, \ldots, p$ and (2.2) holds

$$(3.5) \qquad \int_0^{\pi/2} e^{i(\mu+v)\Theta} \sin^{\mu-1} \operatorname{Cos}^{v-1} \varnothing\; {}_2F_1 \left[\begin{matrix} u, v \\ v \end{matrix}\middle| e^{i\Theta} \operatorname{Cos} \varnothing \right]$$

$$* \; H \left[\begin{matrix} x \operatorname{Sin}^{\lambda} \varnothing \; e^{i\lambda\Theta} \\ y \operatorname{Sin}^{\omega} \varnothing \; e^{i\omega\Theta} \end{matrix}\right] d\Theta$$

$$= e^{i\frac{\pi}{2}\mu} \, \Gamma(\mu)$$

$$* \; H^{l+2,(n,q),*,(m,p)}_{E+2,(A,C)\,F+2,(B,D)} \left[\begin{matrix} x\, e^{i\frac{\pi}{2}\lambda} \\ \\ y\, e^{i\frac{\pi}{2}\omega} \end{matrix} \middle| \begin{matrix} [e : \rho, \varnothing], (g_1 : \lambda, \omega), (g_2 : \gamma, \omega) \\ [a, \alpha] : [c, \gamma] \\ \\ [f : \sigma, \Psi], (h_1 : \lambda, \omega)\; (h_2 : \gamma, \omega) \\ [b, \beta], [d, \delta] \end{matrix} \right]$$

where $g_1 = \mu, g_2 = \mu + v - u - v, h_1 = \mu + v - u, h_2 = \mu + v - v, R\,(\mu) > 0,$ $R\,(v) > 0\, R\,[\mu + v - u - v] > 0, R\,[\mu + \lambda \min (\dfrac{b_i}{\beta_i}) + \omega \min (\dfrac{d_j}{\delta_j})] > 0$ for $i = 1, \ldots, m, j = 1, \ldots, p$ and (2.2) holds.

$$(3.6) \qquad \int_0^{\pi/2} e^{i(\mu+v)\Theta} \operatorname{Sin}^{\mu-1} \varnothing \operatorname{Cos}^{v-1} \varnothing \; {}_2F_1 \left[\begin{matrix} u, v \\ \mu \end{matrix} \middle| e^{i(\Theta - \pi/2)} \operatorname{Sin} \varnothing \right]$$

$$* \; H \left[\begin{matrix} x \operatorname{Cos}^{\lambda} \varnothing \; e^{i\lambda\Theta} \\ y \operatorname{Cos}^{\omega} \varnothing \; e^{i\omega\Theta} \end{matrix}\right] d\varnothing$$

$$= e^{i\frac{\pi}{2}\mu} \, \Gamma(\mu)$$

$$* \; H^{l+2,(n,q),*,(m,p)}_{E+2,(A,C)\,F+2,(B,D)} \left[\begin{matrix} x\, e^{i\frac{\pi}{2}\lambda} \\ y\, e^{i\frac{\pi}{2}\omega} \end{matrix} \middle| \begin{matrix} [e:\rho,\emptyset],(g_1:\lambda,\omega),(g_2:\lambda,\omega) \\ [a,\alpha]:[c,\gamma] \\ [f:\sigma,\Psi],(h_1:\lambda,\omega),(h_2:\lambda,\omega) \\ [b,\beta]:[d,\delta] \end{matrix} \right]$$

where $g_1 = \nu, g_2 = \mu + \nu - u - v, h_1 = \mu + \nu - u, h^2 = \mu + \nu - v;\; R(\mu) > 0$

$$R(\nu) > 0, R(\mu + \nu - u - v) > 0, \quad R\{\nu + \lambda \min(\frac{b_i}{\beta_i}) + \omega \min(\frac{d_j}{\delta_j})] > 0$$

for $i = l, \ldots, m,\; j = 1, \ldots, p$ and (2.2) holds.

(3.7) $$\int_0^{\pi/2} \mathrm{Cos}\, 2\mu\emptyset\, (\mathrm{Cos}\,\emptyset)^{2\nu-2} H \begin{bmatrix} x\,(\mathrm{Cos}\,\emptyset)^{2\lambda} \\ y\,(\mathrm{Cos}\,\emptyset)^{\omega 2} \end{bmatrix} d\emptyset$$

$$= \frac{\pi}{2^{2\nu-1}} H^{l+1,(n,q),*\,(m,p)}_{E+1,(A,C),F+2,(B,D)} \left[\begin{matrix} \frac{x}{2^{2\omega}} \\ \frac{y}{2^{2\lambda}} \end{matrix} \middle| \begin{matrix} [e:\rho,\emptyset],(2\nu-1;2\lambda,2\omega) \\ [a,\alpha];[c,\gamma] \\ [f:\sigma\Psi],(\nu \pm \mu;\lambda,\omega) \\ [b,\beta];[d,\delta] \end{matrix} \right]$$

where λ and ω are positive integers, $R(\nu) > \frac{1}{2}$,

$R[\nu + \lambda \min(\frac{b_i}{\beta_i}) + \omega \min(\frac{d_j}{\delta_j})] >$ — for $i = 1, \ldots, m,\; j = 1, \ldots, p$ and (2.2)
and (2.2) holds.

(3.8) $$\int_0^{\pi} (\mathrm{Sin}\,\emptyset)^{\mu-1} e^{-\nu\Theta} H \begin{bmatrix} x\,(\mathrm{Sin}\,\emptyset)^{2\lambda} \\ y\,(\mathrm{Sin}\,\emptyset)^{2\omega} \end{bmatrix} d\emptyset$$

$$= \frac{\pi e^{-\frac{\pi}{2}\nu}}{2^{\mu-1}} H^{l+1,(n,q),*,(m,p)}_{E+1,(A,C),F+2,(B,D)} \left[\begin{matrix} \frac{x}{2^{2\lambda}} \\ \frac{y}{2^{2\omega}} \end{matrix} \middle| \begin{matrix} [e:\rho,\emptyset],(\mu;2\lambda,2\omega) \\ [a,\alpha];[c,\gamma] \\ [f;\sigma,\Psi],(\mu \pm i\nu + 1;\lambda,\omega) \\ [b,\beta],[d,\delta] \end{matrix} \right]$$

where λ and ω are positive integers, $R(\mu) > 0$,
$R[\mu + 2\lambda \min(\frac{b_i}{\beta_i}) + 2 \min(\frac{d_j}{\delta_j})] > 0$ for $i = 1, \ldots m, j = 1, \ldots, p$
and 2.2) holds.

Proof of Result 3.1. Consider the integral

$$(3.9) \qquad I = \int_0^1 x^{\mu-1} (t-x)^{\nu-1} {}_rF_s \left[\begin{matrix} (u_r) \\ \\ (v_s) \end{matrix} \middle| Px^k \right] H \left[\begin{matrix} yx^{\lambda_1} (t-x)^{\omega_1} \\ \\ zx^{\lambda_2} (t-x)^{\omega_2} \end{matrix} \right] dx$$

On expressing $H \left[\begin{matrix} yx^{\lambda_1} (t-x)^{\omega_1} \\ \\ zx^{\lambda_1} (t-x)^{\omega_2} \end{matrix} \right]$ in terms of double contour integral by (2.1) and then interchanging the order of integration, we obtain

$$I = \frac{1}{(2\pi i)^2} \int_{-i\infty}^{i\infty} \int_{-i\infty}^{i\infty} x_1(\xi)\, x_2(\eta)\, x_3(\xi,\eta)\, y^{-\xi} z^{-\eta}$$

$$* \int_0^1 x^{(\mu-\lambda_1 \xi-\lambda_2\eta)-1} (t-x)^{(\nu-\omega_1 \xi-\omega_2\eta)-1} \quad {}_rF_s \left[\begin{matrix} (u_r) \\ \\ (v_s) \end{matrix} \middle| Px^k \right] dx\, d\xi d\eta$$

On evaluating the inner integral with the help of the result[8, (4), p. 104] and expressing the generalized hypergeometric function in terms of its equivalent series[8, (4), p. 73] and using the modified form of Guass multiplication theorem[8, p. 24]

$$(\alpha)_{nk} = k^{nk} \prod_{i=1}^{k} \quad \Gamma \quad \frac{\alpha + i - 1}{k} \quad n$$

we obtain

$$(3.11) \qquad \mathrm{I} = t^{\mu+\nu-1} \frac{1}{(2\pi \mathrm{i})^2} \int_{-i\infty}^{i\infty} \int_{-i\infty}^{i\infty} X_1(\xi)\, X_2(\eta)\, X_3(\xi,\eta)$$

$$* \frac{\Gamma(\mu - \lambda_1 \xi - \lambda_3 \eta)\, \Gamma(\nu - \omega_1 \xi - \omega_2 \eta)}{\Gamma[\mu + \nu - (\lambda_1 + \omega_1)\xi - (\lambda_2 + \omega_2)\eta]} \quad [yt^{\lambda_1+\omega_1}]^{-\xi} \{zt^{\lambda_2+\omega_2}]^{-\eta}$$

$$* \sum_{j=0}^{\infty} \frac{(u_1)_j \ldots (u_r)_j}{(v_1)_j \ldots (v_s)_j} \quad \frac{(\mu - \lambda_1 \xi - \lambda_2 \eta)_{jk}}{(\mu + \nu - (\lambda_1 + \omega_1)\xi - (\lambda_2 + \omega_2)\eta)_{jk}} \quad \bullet \quad \frac{(Pt^k)^j}{j!}$$

On interchanging the order of integration and summation in (3.11) and interpreting the result thus obtained with the help of (2.1) the result follows.

Regarding the interchange of the order of integration in (3.9) it is observed that the double integral converges if (2.2) holds, the x-integral converges of $R(\mu) > 0$, $R(\nu) > 0$ and the repeated integral converges if

$$R\left[\mu + \lambda_1 \min\left(\frac{b_i}{\beta_i}\right) + \lambda_2 \min\left(\frac{d_i'}{\delta_i'}\right)\right] > 0 \text{ and}$$

$$R\left[\nu + \omega_1 \min\left(\frac{b_i}{\beta_i}\right) + \lambda_2 \min\left(\frac{d_i'}{\delta_i'}\right)\right] > 0 \text{ for } i = 1, \ldots, m \text{ and } i = 1, \ldots, p.$$

Hence the interchange of order of integration is justified by de la Vallee Pansin theorem [2, p. 504].

Since $r \leq s$, the series in (3.11) is absolutely convergent and the term by integration is permissible by virtue of theorem B of Bromwich [2, p. 506].

The results (3.2) to (3.8) can be proved in the same way on applying the results [10,(5), p. 104], [6,(3.1.22), (3.1.23), (3.1.18) and (3.1.19)], {7, Ex 95, p. 304] and [9, p. 159].

4. Particular Cases. It is easy to verify that by a suitable choice of constants and parameters the results (3.1), (3.2), (3.7) and (3.8) reduces to the result obtained by Goyal [3, (2,1) and (2.2)] and Srivastava {12, (2.1) and (2.2)] respectively.

In general on using the properties (2.4) and (2.5) of H-function in the result (3.1) to (3.8) and assigning particular values to λ and ω we can evaluate many integrals involving Agarwal G-function of two variables or product of two H-functions of single variables.

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