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On the H-function transform of two variables II

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ABSTRACT

The present study deals with elaborate proof of the Uniqueness theorem for generalised H-function transform of two variables recently introduced by the author. A lemma is also established in support of the theorem. Later another more theorem has been proved and its various special cases are discussed. The results thus obtained seem to non-existent in the literature.

1. INTRODUCTION

In my recent work on integral transforms [5] (in press), an integral transform involving the H-function of two variables is defined. This transform [5] is a generalisation of the transform introduced by Gupta and Mittal [2], and is expressed by the integral equation:

$$\Phi(s,s_1) = ss_1 \int_0^\infty \int_0^\infty H^{n,v_1,v_2,m_1,m_2}_{p,(t:t^1),\,o,\,(V:V^1)} \left[\begin{array}{c|l} sx & (a_i,\alpha_i)_{1,p} \\ & (\gamma_i,r_i)_{1,t};\ (\gamma_i^1,r_i^1)_{1,t^1} \\ s_1 y & (b_i,\beta_i)_{1,V};\ (b_i^1,\beta_i^1)_{1,V^1} \end{array} \right] \times f(x,y)\, dx\, dy, \tag{1.1}$$

where the H-function of two variables, introduced by Verma [4] is given as

$$H^{n,v_1,v_2,m_1,m_2}_{p,(t:t^1),\,\beta,\,(V:V^1)} \left[\begin{array}{c|l} & (a_i,\alpha_i)_{1,p} \\ x & (\gamma_i,r_i)_{1,t};\ (\gamma_i^1,r_i^1)_{1,t^1} \\ y & (\delta_i,d_i)_{1,\,p} \\ & (b_i,\beta_i)_{1,V};\ (b_i^1,\beta_i^1)_{1,V^1} \end{array} \right] =$$

$$= (2\pi i)^{-2} \int_{-i\infty}^{i\infty} \int_{-i\infty}^{i\infty} \Phi\{A(\xi+\eta)\}\, \psi\,(B\xi, c\eta)\, x^{\xi} y^{\eta}\, d\xi d\eta,$$

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$$\Phi\{A(\xi+\eta)\} = \prod_1^n \Gamma(1-a_j+\alpha_j\xi+\alpha_j\eta)\{\prod_{n+1}^p \Gamma(a_j-\alpha_j\xi-\alpha_j\eta)\prod_1^s \Gamma(\delta_j+d_j\xi+d_j\eta\}^{-1},$$

$$\psi(B\xi,c\eta) = \prod_1^{m_1}\Gamma(b_j-\beta_j\xi)\prod_1^{v_1}\Gamma(\gamma_j+r_j\xi)\prod_1^{m_2}\Gamma(b_j^1-\beta_j^1\eta)\prod_1^{v_2}\Gamma(\gamma_j^1+r_j^1\eta) \times$$

$$\times \{\prod_{m_1+1}^{V}\Gamma(1-b_j+\beta_j\xi)\prod_{v_1+1}^{t}\Gamma(1-\gamma_j-r_j\xi)\prod_{m_2+1}^{V^1}\Gamma(1-b_j^1+\beta_j^1\eta)\}^{-1} \times$$

$$\times \{\prod_{v_2+1}^{t^1}\Gamma(1-\gamma_j^1-r_j^1\eta)\}^{-1}$$

The following notations will be used throughout the present paper:

$(a_i+b,\alpha_i)_{m,n}$, $(n>m)$ will represent $n-m+1$ pairs

(a_m+b,α_m), $(a_{m+1}+b,\alpha_{m+1})$, $\ldots$, (a_n+b,α_n).

Thus, $(a_i,\alpha_i)_{1,n}$ will stand for n pairs (a_1,α_1), $\ldots$, (a_n,α_n). Also, for $(a_i)_p$ we shall denote by (a_p) which will represent $a_1, \ldots, a_p$.
The following results of the author[4-5] will be required in the proofs of the theorems.

$$H^{n,v_1,v_2,m_1,m_2}_{p,(t:t^1),s,(V:V^1)}\left[\begin{array}{c|l} & (a_i,1)_{1,p} \\ x & (\gamma_i,1)_{1,t};\ (\gamma_i^1,1)_{1,t^1} \\ y & (\delta_i,1)_{1,s} \\ & (b_i,1)_{1,V};\ (b_i^1,1)_{1,V^1}\end{array}\right] \tag{1.2}$$

$$= G^{n,v_1,v_2,m_1,m_2}_{p,(t:t^1),s,(V:V^1)}\left[\begin{array}{c|l} & (ap) \\ x & (\gamma_t);\ (\gamma_{t^1}^1) \\ y & (\delta p) \\ & (b_V);\ (b^1_{V^1})\end{array}\right],$$

where the functions on the right of (1.2) is the G-function of two variables.[1]

$$H^{0,0,0,m_1,m_2}_{0,(0:0),0,(V:V^1)}\left[\begin{array}{c|l} & \cdots \\ x & \cdots\ \ \cdots \\ y & \cdots \\ & (b_i,\beta_i)_{1,V};\ (b_i^1,\beta_i^1)_{1,V^1}\end{array}\right] \tag{1.3}$$

$$= H^{m_1,0}_{0,V}\left[\, x \,\middle|\, \begin{array}{l}\cdots \\ (b_i,\beta_i)_{1V}\end{array}\right] H^{m_2,0}_{0,V^1}\left[\, y \,\middle|\, \begin{array}{l}\cdots \\ (b_i^1,\beta_i^1)_{1,V^1}\end{array}\right].$$

$$G^{\alpha,\beta}_{\gamma,\delta}\left[x \,\middle|\, \begin{matrix}(a\gamma)\\ (b\delta)\end{matrix}\right] = \{(2\pi)\ (1-s)\ (\alpha+\beta-\frac{\gamma}{2}-\frac{\delta}{2})\} \ \times$$

$$\times \ \{p(\sum_1^{\delta} b_i-\sum_1^{\gamma} a_i-\frac{\gamma}{2}-\frac{\delta}{2}+1) \ \times \qquad (1.4)$$

$$\times \ G^{s\alpha,s\beta}_{s\gamma,s\delta}\left[\begin{matrix} ss(\gamma-\delta)\\ xs\end{matrix} \,\middle|\, \begin{matrix}\Delta(a_1,s),\ldots,\Delta(a_\gamma,s)\\ \Delta(b_1,s),\ldots,\Delta(b_\delta,s)\end{matrix}\right],$$

where $\Delta(a,s)$ denotes $a/s,\ (a+1)/s,\ \ldots,\ (a+s-1)/s$.

$$H^{m,n}_{p,V}\left[x \,\middle|\, \begin{matrix}(a_i,\alpha_i)_{1,p}\\ (b_i,\beta_i)_{1,V}\end{matrix}\right] = c\,H^{m,n}_{p,V}\left[x^c \,\middle|\, \begin{matrix}(a_i,c\alpha_i)_{1,p}\\ (b_i,c\beta_i)_{1,V}\end{matrix}\right], c>0. \qquad (1.5)$$

$$H^{o,v_1,v_2,m_1,m_2}_{o,(t:t^1),o,(V:V^1)}\left[\begin{matrix} x\\ \\ y\end{matrix} \,\middle|\, \begin{matrix}\cdots\\ (\gamma_i,r_i)_{1,t};\ (\gamma_i^1,r_i^1)_{1,t^1}\\ \cdots\\ (b_i,\beta_i)_{1,V};\ (b_i^1,\beta_i)_{1,V^1}\end{matrix}\right]$$

$$(1.6)$$

$$= H^{m_1,v_1}_{t,V}\left[x \,\middle|\, \begin{matrix}(1-\gamma_i,r_i)_{1,t}\\ (b_i,\beta_i)_{1,V}\end{matrix}\right] H^{m_2,v_2}_{t^1,V^1}\left[y \,\middle|\, \begin{matrix}(1-\gamma_i^1,r_i^1)_{1,t^1}\\ (b_i^1,\beta_i^1)_{1,V^1}\end{matrix}\right].$$

$$G^{o,v_1,v_2,m_1,m_2}_{o,(t:t^1),o,(V:V^1)}\left[\begin{matrix} x\\ y\end{matrix} \,\middle|\, \begin{matrix}\cdots\\ (\gamma_t);\ (\gamma_{t^1}^1)\\ \cdots\\ (b_V);\ (b^1{}_{V^1})\end{matrix}\right] = G^{m_1,v_1}_{t,V}\left[x \,\middle|\, \begin{matrix}(1-\gamma_t)\\ (b_V)\end{matrix}\right] \times$$

$$(1.7)$$

$$\times\ G^{m_2,v_2}_{t^1,V^1}\left[y \,\middle|\, \begin{matrix}(-\gamma_{t^1}^1+1)\\ (b^1{}_{V^1})\end{matrix}\right].$$

2. THEOREMS

We now prove the following lemma in support of the uniqueness theorem for the transform given by (1.1).

LEMMA. If,

$$\int_0^\infty\int_0^\infty H^{n,v_1,v_2,m_1,m_2}_{p,(t:t^1),o,(V:V^1)}\left[\begin{matrix} st\\ \\ s_1t_1\end{matrix} \,\middle|\, \begin{matrix}(a_i,\alpha_i)_{1,p}\\ (\gamma_i,r_i)_{1,t};\ (\gamma_i^1,r_i^1)_{1,t^1}\\ (b_i,\beta_i)_{1,V};\ (b_i^1,\beta_i^1)_{1,V^1}\end{matrix}\right] f(t,t_1)\,t^p t_1^{p^1} dt\,dt_1 = 0$$

$$(2.1)$$

then

$$f(t,t_1) \equiv 0, \tag{2.2}$$

provided that $f(t,t_1)$ is continuous in $t,t_1 \geqslant 0$, $f(t,t_1) = 0$ (t, t_1) for t,t_1 small,

$$A = \sum_1^n (\alpha_j) - \sum_{n+1}^p (\alpha_j) + \sum_1^{m_1} (\beta_j) - \sum_{m_1+1}^V (\beta_j) + \sum_1^{v_1} (r_j) - \sum_{v_1+1}^t (r_j) > 0;$$

$$B = \sum_1^n (\alpha_j) - \sum_{n+1}^p (\alpha_j) + \sum_1^{m_2} (\beta_j^1) - \sum_{m_2+1}^{V^1} (\beta_j^1) + \sum_1^{V^2} (r_j^1) - \sum_{v^2+1}^{t^1} (r_j^1) > 0;$$

$R\ (p+\alpha+b_j|\beta_j+1) > 0, j = 1, \ldots, m_1;\ R\ (p^1+\alpha^1+b_j^1|\beta_j^1+1) > 0, j = 1, \ldots, m_2;$

$|\arg s| < \frac{1}{2} A\pi,\ |\arg s_1| < \frac{1}{2} B\pi;$

$R\ (b_i/\beta_i + (1-b_j)/\beta_j) > 0,\ (j = m_1+1, \ldots, v;\ i = 1, \ldots, m_1);$

$R\ (b_i^1/\beta_i^1 + (1-b_j^1)/\beta_j^1) > 0,\ (j = m_2+1, \ldots, v^1;\ i = 1, \ldots, m_2);$

$R\ (\ (a_i-1)|\alpha_i - a_j|\alpha_j) < 0,\ (i = 1, \ldots, n,\ j = n+1, \ldots, p);$

$R\ (\gamma_i/r_i - (1-\gamma_j)/r_j) < 0,\ (i = 1, \ldots, v_1;\ j = v_1+1, \ldots, t);$

$R\ (\gamma_i^1/r_i^1 - (1-\gamma_j^1)/r_j^1) < 0,\ (i = 1, \ldots, v_2;\ j = v_2+1, \ldots, t^1).$

PROOF. Multiplying (2.1) by

$$\text{(A)}\ \left\{ s^{n_1-1} s_1^{n_2-1}\ H^{p-n,\,t-v_1,\,t^1-v_2,\,c+V-m_1,\,c_1+V^1-m_2}_{p,(t:t^1),\,o,\,(c+V:c_1+V^1)} \left[\begin{matrix} s(z|c)^{Nc} \\ s_1(z_1|c_1)^{N_1c_1} \end{matrix} \middle| \begin{matrix} R_1, R_2 \\ R_3, R_4;\ R_5, R_6 \\ R_7, R_8, R_9;\ R_{10}, R_{11}, R_{12} \end{matrix} \right] \right.,$$

where $cN > A$, $c_1N_1 > B$, $|\arg z| < \frac{1}{2}\pi(1 - A/Nc)$, $|\arg z_1| < \frac{1}{2}\pi(1 - B/N_1c_1)$,

$R_1 = (1-a_i-\eta_1\alpha_i-\eta_2\alpha_i,\alpha_i)_{n+1,p};\qquad R_2 = (1-a_j-\eta_1\alpha_i-\eta_2\alpha_i,\alpha_i)_{1,n};$

$R_3 = (1-\gamma_i+\eta_1 r_i, r_i)_{v_1+1,t};\qquad R_4 = (1-\gamma_i+\eta_1 r_i, r_i)_{1,v_1};$

$R_5 = (1-\gamma_i^1+\eta_2 r_i^1, r_i^1)_{v_2+1,t^1};\qquad R_6 = (1-\gamma_i^1+\eta_2 r_i^1, r_i^1)_{1,v_2};$

$R_7 = (o,N), (1/c,N), \ldots, (\ (c-1)/c,N);\qquad R_8 = (1-b_i-\eta_1\beta_i,\beta_i)_{m_1+1,V};$

$R_9 = (1-b_i-\eta_1\beta_i,\beta_i)_{1,m_1};\qquad R_{10} = (o,N_1), (1/c_1,N_1), \ldots, ((c_1-1)/c_1,N_1);$

$R_{11} = (1-b_i^1-\eta_2\beta_i^1,\beta_i^1)_{m_2+1,V^1}$; $\quad R_{12} = (1-b_i^1-\eta_2\beta_i^1,\beta_i^1)_{1,m_2}$,

and integrating with respect to s and s_1 between the limits 0 to ∞, we get

$$\int_0^\infty\int_0^\infty s^{n_1-1}s_1^{n_2-1}\, H\left[\begin{matrix} s(z|c)^{Nc} \\ s_1(z_1|c)^{N_1c_1}\end{matrix}\middle|\begin{matrix}\cdots\\\cdots\\\cdots\end{matrix}\right]\times \tag{2.3}$$

$$\left\{\int_0^\infty\int_0^\infty H^{n,v_1,v_2,m_1,m_2}_{p,(t:t^1),o,(V:V^1)}\left[\begin{matrix} st \\ s_1t_1\end{matrix}\middle|\begin{matrix}(a_i,\alpha_i)_{1,p}\\(\gamma_i,r_i)_{1,t};\ (\gamma_i^1,r_i^1)_{1,t^1}\\(b_i,\beta_i)_{1,V};\ (b_i^1,\beta_i)_{1,V^1}\end{matrix}\right] t^{\rho}t_1^{\rho^1}dt\,dt_1\right\} ds\,ds_1 = 0,$$

where the parameters of one of the H-function is shown as in (A).
On interchanging the order of integration in (2.3), which is justified due to absolute convergence of the integrals under the conditions imposed, and evaluating the s and s_1—integral, we get

$$\int_0^\infty\int_0^\infty t^{\rho-n_1}t_1^{\rho^1-n_2}\times \tag{2.4}$$

$$\times H^{o,o,o,c,c_1}_{o,(o:o),o,(c:c_1)}\left[\begin{matrix}(z|c)^{Nc}t^{-1}\\(z_1|c_1)^{N_1c_1}t_1^{-1}\end{matrix}\middle|\begin{matrix}\cdots\\(o,N),\ldots,(c-1)/c,N);\\(o,N_1),\ldots,(c_1-1)/c_1,N_1)\end{matrix}\right] f(t,t_1)\,dt\,dt_1 = 0.$$

Now the H-function of two variables in (2.4) reduces to a product of Meijer's G-functions, by applying (1.2), (1.7) and (1.5), its value being equal to

$$(NN_1)^{-1}\ G^{o,o,o,c,c_1}_{o,(o:o),o,(c:c_1)}\left[\begin{matrix}(z/c)^{c}t^{-1|N}\\(z_1/c_1)^{c_2}t_1^{-1|N_1}\end{matrix}\middle|\begin{matrix}\cdots\\\cdots\\(o,1/c,\ldots,\frac{c-1)}{c};\ (o,1/c_1,\ldots,\frac{c_1-1)}{c_1}\end{matrix}\right]$$

$$= (NN_1)^{-1}\ G^{c,o}_{o,c}\left[(z/c)^{c}t^{-1|N}\ \middle|\ o,1/c,\ldots,(c\text{-}1)/c\right] \tag{B}$$

$$\times\ G^{c_1,o}_{o,c_1}\left[(z_1/c_1)\,t_1^{-1|N_1}\ \middle|\ o,1/c_1,\ldots,(c_1-1)/c_1\right]$$

The expression (B) can be reduced with the help of (1.4) to the following expression:

$$(NN_1)^{-1}c^{-\frac{1}{2}}\,c^{-\frac{1}{2}}\,(2\pi)^{-\frac{1}{2}(c+c_1-2)}\quad G^{1,0}_{0,1}\left[zt^{-1/Nc} \,\middle|\, \cdots \right] \times$$

$$G \times G^{1,0}_{0,1}\left[z_1 t_1^{-1/N_1 c_1} \right]. \qquad \text{(C)}$$

Hence (2.4) reduces to the form

$$\int_0^\infty \int_0^\infty t^{p-n_1}\, t_1^{p^1-n_2} f(t,t_1)\ \exp\left(-zt^{-1/Nc} - z_1 t_1^{-1/N_1c_1}\right) dt\, dt_1 = 0 \qquad (2.5)$$

or

$$\int_0^\infty \int_0^\infty t^{-(p-n_1)Nc-Nc-1}\ t_1^{-(p^1-n_2)N_1c_1-N_1c_1-1}\ f(t^{-Nc}, t_1^{-N_1c_1}) \times \exp(-zt - z_1t_1)\, dt\, dt_1 = 0. \qquad (2.5)^1$$

In the light of Lerch's THEOREM [3], we get

$$f(t^{-Nc}, t_1^{-N_1c_1}) \equiv 0. \qquad (2.6)$$

Now from (2.6) it follows that $f(t,t_1) \equiv 0$.

The Uniqueness Theorem.

If $f_1(t,t_1)$ and $f_2(t,t_1)$ are continuous in $t \geqslant 0$, $t_1 \geqslant 0$, and

$$\int_0^\infty \int_0^\infty H^{n,v_1,v_2,m_1,m_2}_{p,(t:t^1),\,o,\,(V:V^1)}\left[\begin{matrix} st \\ s_1t_1 \end{matrix}\,\middle|\, \begin{matrix} (a_i,\alpha_i)_{1,p} \\ (\gamma_i,r_i)_{1,t};\ (\gamma_i^1,r_i^1)_{1,t^1} \\ (b_i,\beta_i)_{1,V};\ (b_i^1,\beta_i^1)_{1,V^1} \end{matrix}\right] f_1(t,t_1)\, dt\, dt_1$$

$$= \int_0^\infty \int_0^\infty H^{n,v_1,v_2,m_1,m_2}_{p,(t:t^1),\,o,\,(V:V^1)}\left[\begin{matrix} st \\ s_1t_1 \end{matrix}\,\middle|\, \begin{matrix} (a_i,\alpha_i)_{1,p} \\ (\gamma_i,r_i)_{1,t};\ (\gamma_i^1,r_i^1)_{1,t^1} \\ (b_i,\beta_i)_{1,V};\ (b_i,\beta_i^1)_{1,V^1} \end{matrix}\right] f_2(t,t_1)\, dt\, dt_1, \qquad (2.7)$$

both integrals converging, then

$$f_1(t,t_1) \equiv f_2(t,t_1). \qquad (2.8)$$

PROOF. From (2.7), we get

$$\int_0^\infty \int_0^\infty H^{n,v_1,v_2,m_1,m_2}_{p,(t:t^1),o,(V:V^1)} \left[\begin{matrix} st \\ s_1t_1 \end{matrix} \middle| \begin{matrix} (a_i,\alpha_i)_{1,p} \\ (\gamma_i,r_i)_{1,t};\ (\gamma_i^1,r_i^1)_{1,t^1} \\ (b_i,\beta_i)_{1,V};\ (b_i^1,\beta_i^1)_{1,V^1} \end{matrix} \right] \{f_1(t,t_1) - f_2(t,t_1)\} dt\, dt_1 = 0.$$

Hence, by above lemma, we obtain

$$f_1(t,t_1) - f_2(t,t_1) \equiv 0$$

or

$$f_1(t,t_1) \equiv f_2(t,t_1).$$

3. In this section we shall establish a theorem relating to the transform (1.1) and the following transform (a special case of (1.1)):

$$G \left\{ f(x,y)\,;\ \begin{matrix} c_1, .., c_r;\ d_1, .., d_r, \bar{\xi} \\ c_1^1, .., c_{r^1}^1;\ d_1^1, .., d_{r^1}^1, \eta \end{matrix} ;\ s, s_1 \right\} \tag{3.1}$$

$$= ss_1 \int_0^\infty \int_0^\infty G^{0,0,0,r+1,r^1+1}_{0,(r:r^1),0,(r+1:r^1+1)} \left[\begin{matrix} sx \\ s_1y \end{matrix} \middle| \begin{matrix} \cdots \\ D_1;\ D_2 \\ C_1, \bar{\xi};\ C_2, \bar{\eta} \end{matrix} \right] f(x,y)\, dx\, dy,$$

where $D_1 = 1 - c_1 - d_1, \ldots, 1 - c_r - d_r$; $D_2 = 1 - c_1^1 - d_1^1, \ldots, 1 - c_{r^1}^1 - d_{r^1}^1$;
$C_1 = d_1, \ldots, d_r$; $C_2 = d_1^1, \ldots, d_{r^1}^1$.

THEOREM. If

$$\Phi(s,s_1) = ss_1 \int_0^\infty \int_0^\infty H^{n,v_1,v_2,m_1,m_2}_{p,(t:t^1),o,(V:V^1)} \left[\begin{matrix} sx \\ s_1y \end{matrix} \middle| \begin{matrix} (a_i,\alpha_i)_{1,p} \\ (\gamma_i,r_i)_{1,t};\ (\gamma_i^1,r_i^1)_{1,t^1} \\ (b_i,\beta_i)_{1,V};\ (b_i^1,\beta_i^1)_{1,V^1} \end{matrix} \right] h(x,y)\, dx\, dy \tag{3.2}$$

then

$$s^{\rho+\sigma} s_1^{\rho^1+\sigma_1} \Phi(s^{-\sigma}, s_1^{-\sigma_1}) = G \left\{ t^{\rho-} t_1^{-\rho^1} \left[\int_0^\infty \int_0^\infty F(tx,t_1y)\, h(x,y)\, dx\, dy \right];\ \begin{matrix} c_1, .., c_r;\ d_1, .., d_r, \bar{\xi} \\ c_1^1, .., c_{r^1}^1;\ d_1^1, .., d_{r^1}^1, \bar{\eta} \end{matrix} ;\ s, s_1 \right\}, \tag{3.3}$$

where

$$F(tx, t_1y) = \tag{3.4}$$

$$H^{n,v_1+r,v_2+r^1,m_1,m_2}_{p,(t+r:t^1+r^1),\, o,\, (V+r+1:V^1+r^1+1)} \left[\begin{matrix} xt^{\sigma} \\ yt_1^{\sigma_1} \end{matrix} \middle| \begin{matrix} A_1 \\ A_2, A_3; A_4, A_5 \\ A_6, A_7, A_8; A_9, A_{10}, A_{11} \end{matrix} \right],$$

provided that

$h(x,y) = 0\,(x^{\alpha} y^{\alpha^1})$, for x,y small,

$R(s) > 0, R(s_1) > 0, A - \sigma > 0, B - \sigma_1 > 0,$

(where $A = \sum_1^n (\alpha_j) - \sum_{n+1}^p (\alpha_j) + \sum_1^{m_1} (\beta_j) - \sum_{m_1+1}^{v} (\beta_j) + \sum_1^{v_1} (r_j) - \sum_{v_1+1}^{t} (r_j) > 0;$

$B = \sum_1^n (\alpha_j) - \sum_{n+1}^p (\alpha_j) + \sum_1^{m_2} (\beta_j) - \sum_{m_2+1}^{V^1} (\beta_j^1) + \sum_1^{v_2} (r_j^1) - \sum_{v_2+1}^{t^1} (r_j^1) > 0,$

$A_1 = (a_i, \alpha_i)_{1,p}; \quad A_2 = (1 - p + d_i + c_i, \sigma)_{1,r}; \quad A_3 = (\gamma_i, r_i)_{1,t};$

$A_4 = (1 - p^1 + d_i^1 + c_i^1, \sigma_1)_{1,r^1}; \quad A_5 = (\gamma_i^1, r_i^1)_{1,t^1};$

$A_6 = (b_i \beta_i)_{1,v}; \quad A_7 = (p - d_i, \sigma)_{1,r};$

$A_8 = (\sigma - \xi, \sigma); \quad A_9 = (b_i^1, \beta_i^1)_{1,v^1};$

$A_{10} = (p^1 - d_i^1, \sigma_1)_{1,r}; \quad A_{11} = (p^1 - \bar{\eta}, \sigma_1),$

$R(\alpha + 1 + b_i | \beta_i) > 0, i = 1, 2, \ldots, m_1; R(\alpha^1 + 1 + b_i^1 | \beta_i^1) > 0,$

i $= 1, \ldots, m_2$ and the G-function transform of

$$t^{-p}\, t_1^{-p^1} \int_0^{\infty} \int_0^{\infty} F(tx, t_1y)\, h(x,y)\, dx\, dy, \text{ exists.}$$

PROOF. As,

$$\Phi(s, s_1) = ss_1 \int_0^{\infty} \int_0^{\infty} H^{n,v_1,v_2,m_1,m_2}_{p,(t:t^1),\, o,\, (V:V^1)} \left[\begin{matrix} sx \\ s_1y \end{matrix} \middle| \begin{matrix} (a_i, \alpha_i)_{1,p} \\ (\gamma_i, r_i)_{1,t}; (\gamma_i^1, r_i^1)_{1,t^1} \\ (b_i, \beta_i)_{1,v}; (b_i^1, \beta_i^1)_{1,v^1} \end{matrix} \right] h(x,y)\, dx\, dy;$$

we have

$$s^{p+\sigma} s_1^{p_1+\sigma_1} \Phi (s^{-\sigma}, s_1^{-\sigma_1}) =$$

$$s^p s_1^{p_1} \int_0^\infty \int_0^\infty H^{n,v_1,v_2,m_1,m_2}_{p,(t:t^1),o,(V:V^1)} \left[\begin{matrix} xs^{-\sigma} \\ ys_1^{-\sigma_1} \end{matrix} \middle| \begin{matrix} (a_i,\alpha_i)_{1,p} \\ (\gamma_i,r_i)_{1,t};\ (\gamma_i^1,r_i^1)_{1,t^1} \\ (b_i,\beta_i)_{1,V};\ (b_i^1,\beta_i^1)_{1,V^1} \end{matrix} \right] h(x,y)\, dx\, dy. \tag{3.5}$$

Also,

$$s^p s_1^{p^1} H^{n,v_1,v_2,m_1,m_2}_{p,(t:t^1),o,(V:V^1)} \left[\begin{matrix} xs^{-\sigma} \\ ys_1^{-\sigma_1} \end{matrix} \middle| \begin{matrix} (a_i,\alpha_i)_{1,p} \\ (\gamma_i,r_i)_{1,t};\ (\gamma_i^1,r_i^1)_{1,t^1} \\ (b_i,\beta_i)_{1,V};\ (b_i^1,\beta_i^1)_{1,V^1} \end{matrix} \right]$$

$$= ss_1 \int_0^\infty \int_0^\infty t^{-p}\, t^{-p^1} H^{0,0,0,r+1,r+1}_{o,(r:r^1),o,(r+1:r^1+1)} \left[\begin{matrix} st \\ s_1 t_1 \end{matrix} \middle| \begin{matrix} \cdots \\ 1-c_1-d_1,\ldots,1-c_r-d_r; \\ 1-c_1^1-d_1^1,\ldots,1-c_{r^1}^1-d_{r^1}^1 \\ d_1,\ldots,d_r,\bar{\xi};\ d_1^1,\ldots,d_{r^1}^1,\bar{\eta} \end{matrix} \right] \times$$

$$\times\ F(tx,t_1y)\, dt\, dt_1. \tag{3.6}$$

Now substituting the value of $H\left[\begin{matrix} xs^{-\sigma} \\ ys_1^{-\sigma_1}\end{matrix} \middle| \begin{matrix}\cdots\\ \cdots \\ \cdots\end{matrix}\right]$ from (3.6) in (3.5), changing the order of integration therein (which is justified on account of the absolute convergence of the integrals involved under the conditions stated) and finally applying (3.1), one can obtain the result.

The integral transforms involved in this theorem are general in nature. A number of theorems relating the Laplace transform, Hankel transform and its various generalisations, follow as special cases of this theorem. The integral transform (1.1) includes the generalised transform due to Bhise, Meijer, Varma, etc., as special cases as indicated by Bhise [6].

By taking $n = 0$, $p = 0$ in (3.2) and (3.4), using (1.6) in (3.2) and in (3.4), and finally using (1.7) in (3.1), this theorem reduces to a generalisation of the theorems established by Gupta and Mittal.[2] Thus, in turn the results may prove of general interest. Since a large variety of functions that occur frequently in problems of analysis and mathematical physics are only specialised or limiting forms of the kernel used in the present transform, our findings may be of great importance.

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