

Noctilucent Clouds

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In the new scenario, NASA invited to find the math model that shows the time to observe those clouds, but also to explain why, when and where is it possible to observe them.

OLD PBL model scenario:

Polar mesospheric clouds (PMCs), also known as noctilucent clouds (NLCs), are bright cloudlike atmospheric phenomena visible in a deep twilight. The name means roughly, “night shining” in Latin. Unlike more common clouds that form up to 5 miles above the surface of the Earth, the NLCs are 50 miles high in a layer of the atmosphere called the mesosphere. They are most often observed in the summer months at latitudes between 50° and 60° north and south of the equator. In recent years, however, several people have reported seeing NLCs at lower latitudes, even as low as 40°N in the continental United States.

NASA invited to find the mathematical model of limit that shows the time we have to observe those beautiful clouds. When I applied the mentioned scenario 2 semesters ago, students designed different math models but all of them were limited to merely illustrate observation time.

In the new scenario, NASA invited to find the math model that shows the time to observe those clouds, but also to explain why, when and where is it possible to observe them. In my opinion, the new scenario is much better because as the Sun Rays model proves below, finding the mathematical model of limit that shows only time, may give too much direction.

With the new scenario the problem is complex and messier because it can lead the students to decide to investigate not only the observation times of the noctilucent clouds, but also to analyze other elements of the phenomenon, such as the cloud's constitution. Students might even attempt to create a different "math model" that illustrates the composition of noctilucent clouds, the reason of their appearance, and explains why is the electric blue color, why is the shining, why we can see them only at night, etc.

Goals and objectives

Within the new scenario, students' objective is to research and analyze information about noctilucent clouds to have a global and clear context regarding the elements which are involved in this phenomena, in order to have a mathematical interpretation in the form of a model. As a result from that model, students will be able to identify the places where the clouds are seen, find the causes why these clouds can be seen, obtain the observer's angle, as well as being able to predict when and where the observer can see this phenomenon. They will also be capable to find Earth's circumference, radius, the angle cloud-earth-sun, and the angular distance. Besides, they will be able to find Earth's arc length (cloud's time observation), and sector area formula; to find the incidence angle of the sun with respect of the Earth, understanding the translational movement of the Earth around the sun and analyzing if that movement affects the incidence of the sun rays at the North Pole, which make noctilucent clouds visible. Finally, students will be able to understand summer solstice and analyze the constitution of noctilucent clouds as well as the reasons for their presence in the mesosphere, their appearance, their electric blue color, their shine and why we can see them only at night.

As bibliographical research objectives the students must be capable of making a mathematical model that includes antecedents, previous investigation of the problem, hypothesis, objectives, research fundamentals, limitation, and glossary of terms, including fundamental theoretical elements for the problem solution, process that has a logical and coherent sequence.

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As a product with the Math Model we'll be able to recognize the variables that are involved in the phenomenon and to consider the relevance of those variables on the model. We will understand the behavior of the phenomenon while the variables are changing, and the students will be capable to establish conclusions with the obtained information.

Sun Rays Model

For this investigation, students have to explore different books of Analytic Geometry, Trigonometry, Physics and Geography (ellipse formula, angles' laws, angular displacement, arc length, sector, area). They have to investigate scientific vocabulary and research about ray sun incidence angle, investigate how the ray sun incidence angle change with respect the seasons, find and obtain information on the Internet and select that reliable and truthful facts, as well as using the GLOBE atmospheric protocols to collect data online. They need to do collaborative work: to discuss the information, to develop the mathematical model, and to work on the model's presentation using technology.

Sun Rays Model PBL

Polar mesospheric clouds (PMCs), also known as noctilucent clouds (NCLs), are bright cloud like atmospheric phenomena visible in a deep twilight. The highest clouds in Earth's atmosphere are made of frozen water, or ice crystals, just like some of the clouds that appear in the sky every day. Unlike more common clouds that form up to 5 miles above Earth's surface, these clouds are 50 miles high in a layer of the atmosphere called mesosphere.

Scientists are attempting to find out why these clouds form and why they are changing. In particular, they wish to determine if these changes are caused by natural variations in Earth's atmosphere, or if they are influenced by human activities. NASA invited us to create a math model in which we could show possible relationships between relatively recent phenomenon of noctilucent clouds and global warming and climate change.

Goals and objectives

Students will research and analyze information about noctilucent clouds to have a global and clear context about the elements involved in this phenomenon in order to have a mathematical interpretation.

Scientists are attempting to find out why these clouds form and why they are changing. In particular, they wish to determine if these changes are caused by natural variations in Earth's atmosphere, or if they are influenced by human activities. NASA invited us to create a math model in which we could show possible relationships between relatively recent phenomenon of noctilucent clouds and global warming and climate change

It is expected that with this information, students will infer the general composition of the noctilucent clouds, infer the noctilucent cloud density, find the altitude of de clouds, understand the incident sun ray on the Earth, as well as the reflected ray sun. They will be able to find the incidence angle of the sun according to the seasons, to understand the phenomenon of global warming, what the relationship between noctilucent clouds and global warming is and if it exists a relationship between both. Besides, students must be capable to apply the knowledge of reflection's law to a real world problem and to obtain the angles noctilucent cloud's phenomenon reflection and refraction.

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The model has to include sufficient and necessary information about variables to be comprehensive and to be able to analyze the behavior phenomenon and the variables that affect it. Students must be aware that a bibliographical research should be complemented with a field research, and they must be capable of establishing conclusions based on the mathematical model.

As resources, the students need to research elements about noctilucent clouds such as its geographic position, etc. The objective is to have a general concept of noctilucent clouds (where they are? why they are there? which is the scientific information available about this clouds? etc.), and to research about ray sun incidence angle, global warming, and climate change, of course, being reliable to identifying trustful information. Students need to do collaborative work: to discuss the information, to develop the mathematical model, and to work on the model's presentation using technology.

PBL solution and results

The student team formed by Gustavo Javier Bonilla Lara, Alfonso Gafford Soto, Mirna Andrea Silerio Martínez, Laura Fabiola Suárez Hernández and Carlos Israel Montoya Martínez presented the following results as part of the PBL scheme.

1. Noctilucent clouds context

Noctilucent Clouds (NLC) are located over 80 km high in the atmosphere and are mainly constituted by ice. The importance of studying these clouds is that some changes in the atmosphere have appeared. Since these climate changes are not truly understood and since the planet has been reacting to various shifts and changes, these clouds do provide a possible answer. Thus knowing this furthers curiosity to determine what is happening to the planet.

NASA invites us to find the math model that shows, illustrates, and explains why, where, and when it is possible to observe these beautiful clouds. In addition, we should find the relationship between NLC and global warming.

This is the scenario about the math model, what is the temperature and what is the pressure where NLC's are; what is the CO2 concentration in the last years.

2. Temperature as function of height

To find the temperature from the different layers of the atmosphere, mathematical functions need to be obtained because of the temperature changes in each of these layers.

The temperature is in function of the height:

Troposphere: (0 → 10km)

$$f(h) = -15h / 2 + 25$$

Stratosphere: (10 → 50km)

$$f(h) = h/2 + 50$$

Mesosphere: (50 → 90km)

$$f(h) = -8h + 410$$

Thermospere (90 → 380km)

$$f(h) = 11h - 1195$$

Consequently, the temperature where the NLC's is:

Mesosphere

$$f(h) = -8h + 410$$

$$f(85) = -8(85) + 410$$

$$f(85) = -270 \text{ }^\circ\text{C}$$

Therefore, cold temperatures are the first condition for the creation of NLC's.

3. Atmospheric pressure as a function of height

The atmospheric pressure where the noctilucent clouds are needs to be obtained in order to estimate the development of these clouds.

The atmospheric pressure is in function of height:

$$f(h) = 1035 e^{(-0.12)(h)}$$

(Where height is in Km and the atmospheric pressure is in isobars)

Consequently, in order to find the atmospheric pressure 85 kilometers up in the atmosphere where the NLC are, it is necessary to substitute the height in this function:

$$f(h) = (1035) e^{(-.12)(h)}$$

$$f(85) = (1035) e^{(-.12)(85)}$$

$$f(85) = .038471 \text{ isobars}$$

The atmospheric pressure is the second condition for the creation of NLC's.

4. CO2 Concentration

The concentration of CO2 has increased in the last fifty years because of human conventional activities: aerosol use, vehicle emissions, and industrial

pollutant, etc. From the beginning of the last century, the emissions of greenhouse gases had been emitted in the atmosphere causing global warming.

Each year the concentration of CO₂ increases on average of 3.7%.

The particles are the third condition for the creation of NLC's. This can be represented by the following math regression:

$$f(x) = .0068333202723201 x^2 - .295077599671x + 300.4193$$

where "x" represents years

Years	CO ₂ concentration	x - value
1880	293	0
1900	297	20
1910	300	30
1920	305	40
1930	307	50
1940	308	60
1950	311	70
1960	315	80
1970	323	90
1980	339	100
1990	355	110
2000	368	120
2010	383	130
2020	405	140
2030	432	150
2040	461	160
2050	493	170
2060	524	180
2070	554	190
2080	582	200
2090	607	210
2100	630	220

5. Time of NLC's visibilit

$$\lim_{x \rightarrow e} \frac{(\text{time}) (e)}{360^\circ}$$

Introduction and Precedents

Nowadays, the entire world faces a new phenomenon that is not still established as dangerous for the life in it. This fascinating strange phenomenon refers to the noctilucent clouds, which as their name affirms, they can be perceived at night, a few minutes later the sunset. These kind of specific clouds are different than any other conventional cloud, and it is their capacity of reflecting a shiny beautiful electric blue color at the earth and that they are extremely high on the mesosphere (approximately 80 km from the sea level). Experts have concluded that this is because of their particular composition. There are two theories that support the existence and the composition of this wonderful clouds, one assures that they are made from cosmic dust that entered to the atmosphere, and the other one establish that they exist thanks to the global warming, which means that they are made of contaminants. The last one has been considerate as the most credible of both, but both still alive.

The information that is clearly known about these clouds consists in:

- These clouds can be observed in the north areas of the earth, places near of the North Pole.
- Countries like Alaska, and France, but in the last few years the number of places in which they can be seen has increased considerably, which means that these clouds are spreading over the entire world.
- Noctilucent clouds, which mean “night shining clouds,” can be perceived when the sun is 6 degrees to 16° below the horizon of the observer, and it last for 30 minutes to 1 hour approximately.
- They started to be seen in 1800 approximately.
- The actual localization of them is almost in the border of the atmosphere with the outer space, in the mesosphere, which is a dry cold cap, and it has conditions to convert everything that arrives to this cap, so this supports the idea of the contaminants that are converted into dust un this cap.
- The altitude of these clouds above the ground is from 80 to 100 km.
- They can be seen more easily during the summer.

Objectives

Knowing the mentioned information, *this project consists in determine the observation area considering the rotation and translation movements of earth and the nearness with the sun. At the end, we will be capable to determine exactly the limits on the observation area at any time of the year.* The main goal of finding the

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observation area is to estimate exactly the observation area in all over the world, at any time, hour and specific day of the year. We are going to explain

Hypothesis

“It is possible to determine the limits of the observational area for the noctilucent clouds at any time of the year using the known data and some calculus.”

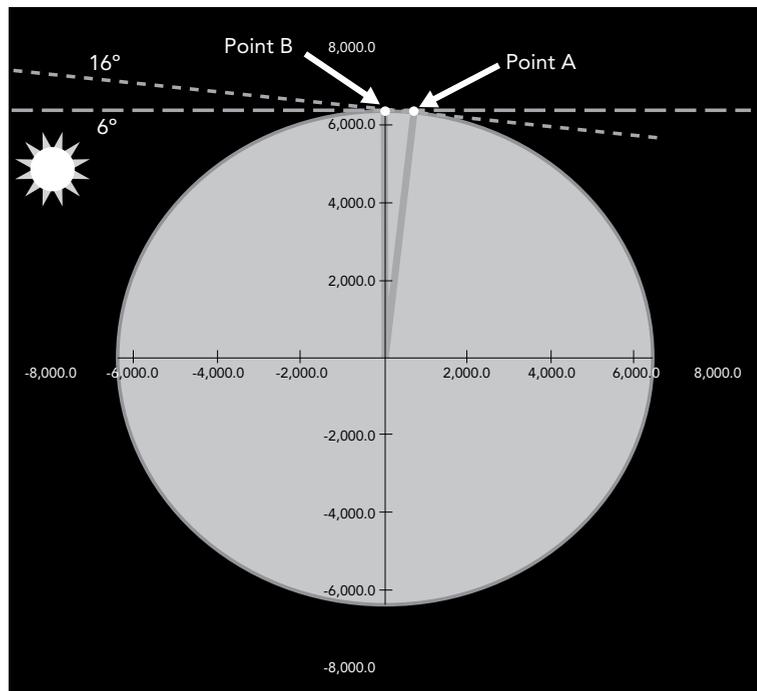
Procedure

The Project is now divided in two parts; the first one is the one that is already finished, and the second part is the one that is going to be made in the future, based in the investigations and proven facts provided by the first part. Here is the explanation:

First Part

Our objectives for this part were to determine the angle of observation in the earth that is made when the sun is from 6° to 16° below the sun. Also to determine the length of the observational area in the earth, the time in which we can see this phenomenon and prove it using calculus, analytic geometry, trigonometry and physics.

For to do this, we used the following sketch:



- We set the earth as a circle in a Cartesian plane with its center at the origin, and we give to the earth the following formula (we used the real dimensions of the earth radius): $x^2+y^2=40678884$
- We determined the observational angle using the properties of the angles knowing that the angle between the two tangents is 10 degrees. The explanation is on the presentation. *Observational angle = 10°*
- We consider one limit of the observational area at $x=0$ because that can be a point on the earth in which the day becomes night. With the circle equation we determine one limit point: $P1(0, 6378)$.
- Using a rule the following rule of three we determined the other limit on the observational area that is on $x=708.67$. We get the other limit point: $P2 (708.67)$.

- We got the equations of the tangent lines at the observational limit points in order to know the two different horizons, and to see that there are really 10° of difference between them. For to do this we derivate the implicit equation of the circle:

$$x^2+y^2=40678884$$

$$y' = -x/y$$

With the derivative we obtained the slope of the two different tangents at the two limit points, and finally we got the equations of the tangents.

Tangent at P1: $y=6378$

Tangent at P2: $y=0.1118x+6259.28$

- We got the arc-length of the circle that represents the perimeter of the observational area. We used the following formula

$$s=(\Theta)(r)$$

S represents the arc-length of the observational perimeter in Km

Θ represents the observational angle in radians

r represents the radius of the earth in Km

$$s = (0.1745 \text{ radians}) (6378 \text{ Km})$$

$$s = 1,113.1736 \text{ Km}$$

- Finally, we got the time in which an observer placed on the point B can see the phenomenon. We used a dimensional analysis:

$$24 \text{ hours} = 360^\circ$$

$$x = 10^\circ$$

$$x = 0.6666 \text{ hours}$$

$$X = 40 \text{ minutes}$$

Second Part

Using the data that we obtained and the data of the rotational and translational movement of the earth, we are going to find the observational limits of the other different clouds and we are going to show its changes while time is passing. We will also prove the distance between the tangents is 10° using the reflection properties of the light. We will also determine the area in which a NLC can be for to be visible. At the end, we will be capable to determine at any day of the year where and how can a NLC be visible.

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General conclusions

With the data presented before, it can be deduced that NLC's are formed because of some climate changes. The CO₂ levels are increasing, the temperature in the mesosphere is too cold, and the atmospheric pressure favors the formation of NLC's.

According to James Russel III, NLC are not causing global warming because the layer of these clouds is too thin to reflect the infrared radiation. However, it is thought that global warming is provoking the existence of NLC's.

A proposed theory: NLC's could be a natural response to global warming; it means that Earth by itself is creating a way to control the damages that humankind has caused. ●

Nubes noctilucentes

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Siguiendo la metodología del aprendizaje basado en problemas, un grupo de alumnos y profesores de preparatoria trabajaron por invitación de la NASA en la búsqueda de un modelo matemático que explicase el fenómeno de las nubes noctilucentes. A diferencia de las nubes comunes formadas a 5 millas sobre la superficie de la tierra, las noctilucentes se ubican en la capa más alta de la mesosfera, 50 millas más arriba, y de acuerdo con los resultados obtenidos, son una respuesta natural de la Tierra ante el cambio climático. A través de procesos matemáticos, los alumnos identificaron en la temperatura, altitud, presión atmosférica y concentración de dióxido de carbono, las variables que influyen en la formación y visibilidad de estos cuerpos gaseosos. ●

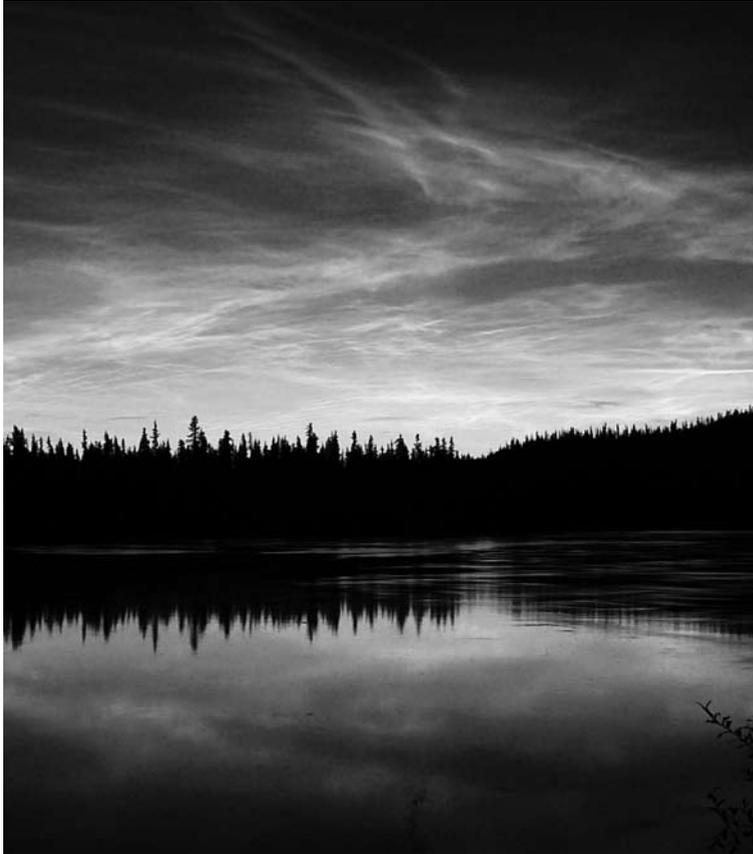


Photo above, taken by David Cartier, Sr. shows a breathtaking display of noctilucent clouds above the waters of the Yukon River, near Whitehorse in the Yukon Territory of Canada. It was captured approximately 2:00 a.m. (local time) July 24, 2007.

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