Observing nanostructures with the Bohr-Heisenberg microscope: a subject for introductory modern physics courses

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In this work we obtain the ground state energy of the infinite potential well using a quantum concept such as the Bohr-Heisenberg microscope prevailing before the advent of the formalization of quantum mechanics and the uncertainty principle. Such energy value is equal to that obtained using formal quantum mechanics. We used this result to estimate the size of novel quantum structures such as the so called nanostructures or quantum wells, currently under study in solid state physics. This idea could be useful in teaching undergraduate introductory modern physics courses.

Keywords: Modern physics; Heisenberg microscope; potential well; nanostructures.

En este trabajo se muestra la energía de estado base de un pozo de potencial infinito, usando un concepto cuántico tal como el del microscopio Bohr-Heisenberg, que prevalecía hasta antes de la formalización de la mecánica cuántica y del principio de incertidumbre. El resultado del valor de la energía es igual al obtenido por medio del uso de mecánica cuántica formal, y es usado para estimar el tamaño de nuevas estructuras cuánticas, tales como las nanoestructuras o los pozos cuánticos, que actualmente se encuentran en el campo de estudio de la física de estado sólido. Esta idea seria de gran utilidad para fines didácticos dentro de los cursos universitarios de física moderna introductoria.

Descriptores: Física moderna; microscopio de Heisenberg; pozo de potencial; nanoestructuras.

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1. Introduction

Before the advent of the formal development of quantum mechanics, a few simple, fundamental problems were worked out exactly. It was the case of the Bohr atomic model for the hydrogen atom. This kind of problem is treated in quantum mechanics and modern physics textbooks prior to the introduction of Schrödinger's equation with which more rigorous and general solutions can be obtained than those using the primitive approach. In this work we obtain the ground state energy of an infinite potential well using pre-quantum mechanics ideas only. We consider a gedanken experiment of a photon incident on a particle being observed through a microscope (the "Bohr-Heisenberg" Microscope). This was conceived by Bohr and for the first time conveyed Heisenberg uncertainty principle, which was later formalized within the framework of quantum mechanics.

In the first studies of quantum mechanics courses a simple expression is used: the relation of the Heisenberg uncertainty principle to obtain information of a complex problem, the ground state of the hydrogen atom for instance.

The calculations presented here to determine the ground state of an infinite potential well are quite simple, and to our knowledge are not found in the literature.

This could stand as another pre-quantum mechanics example to be included in introductory textbooks [1-3] on the subject for students not familiar with the uncertainty principle. Also, it could serve to introduce the concept of an infinite potential well later treated with Schrödinger's equation. We know that most standard text books on quantum mechanics usually discuss three problems with exact analytical solutions: the infinite potential well, the harmonic oscillator and the hydrogen atom [4]. It is possible to find out other problems with an exact analytical solution in the research literature [5,6], one of them an interesting problem of systems with position dependent masses *i.e.* m(x), (m mass and x position) [6].The infinite potential problem is not calculated in a simple form and is not handled in books that we mention above, but appeals to the uncertainty principle because it is reasonable to identify the position uncertainty with the length of the well and the momentum uncertainty through the kinetic energy of the trapped particle, but this gives us only an approximate value. The approach presented here with pre-quantum mechanics ideas, similarly to the Bohr atom in the case of the hydrogen atom, gives us the exact value. This is valuable teaching tool as an introducing in the classroom or a textbook.

As an interesting application of the result obtained, we use it in nanostructures studied at the present time in solid state physics using quantum wells.

2. Theory

The idea of Bohr regarding the measurement of the position and the momentum of a particle, "seeing it" through an optic microscope, served "to deduce" Heisenberg's uncertainty principle and to present it in a graphic way, mainly for didactic purposes.

On the other hand, the Heisenberg's uncertainty principle is presented in elementary physics texts [3] and has been used to obtain the order of magnitude of the ground state of the hydrogen atom.

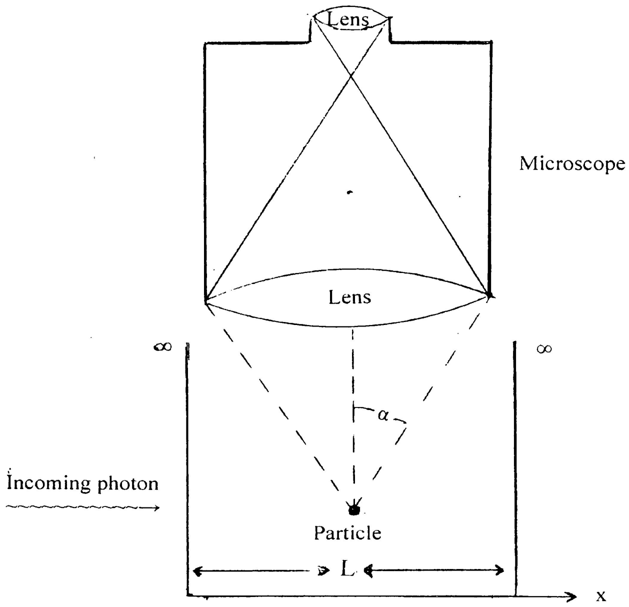

FIGURE 1. Seeing a particle in a potential well using the Bohr-Heisenberg Microscope.

Although with less accuracy, approximate values can be obtained for the energy states on diverse problems. For a one dimensional infinite square well of longitude L, Fig. 1, a gross approximation is obtained, but using the Bohr-Heisenberg microscope, we will obtain the exact value.

The resolving power of a microscope [7] gives us that the separation distance for which two objects can just be resolved is:

$$\Delta x = \frac{\lambda}{2\sin\alpha} \quad (1)$$

where α is the half- angle subtended by the aperture.

Doing the deduction of the ground state of the infinite potential well, suppose a photon is incident upon a particle trapped in a potential well with infinite walls as shown in Fig. 1. An observer looking though an ideal microscope needs, we assume only one photon to view the scene and make measurements. The momentum associated with the photon is given by the De Broglie relationship

$$P = \frac{h}{\lambda} \quad (2)$$

i.e. Planck's constant over the wavelength of the photon. The photon is scattered by the particle within a cone of angle 2α. By conservation of momentum the uncertainty in the momentum of the particle after the scattering must be at least equal to the scattered photon, if the total momentum of the photon is P, then:

$$P_x = P\sin\alpha = \frac{h\sin\alpha}{\lambda} \quad (3)$$

P_x is the x component of the momentum of the scattered photon.

Since the x component of the photon momentum can be known exactly before the collision, the conservation of momentum requires that the trapped particle acquire a momentum of magnitude ΔPx equivalent to the magnitude of the photon's momentum, that is:

$$\Delta Px = \frac{h\sin\alpha}{\lambda} \quad (4)$$

The uncertainty in x is the resolution between points of the observed object, from optics: it is the wavelength of the electromagnetic radiation over sinα of the same angle appearing in Eq. (1).

Thus, the product of the uncertainties in x and *Px* at the moment of observation is

$$\Delta x \Delta Px = \frac{h}{2} \quad (5)$$

This relation is in accordance with the Heisenberg's uncertainty principle.

To calculate the ground state of the particle trapped in a well of width L, we use the uncertainty in Δx equal to L. From (5) the uncertainty in momentum is

$$\Delta Px = \frac{h}{2L}$$

So the ground state energy of the potential well with width L is:

$$E = \frac{(\Delta P_x)^2}{2m} = \frac{h^2}{8mL^2} \quad (6)$$

But the value of Eq. (6) corresponds to the ground state of an infinite potential well [4].

An important consequence of the use of the uncertainty principle is that a particle confined to a small space cannot have zero energy, as is shown by quantum mechanics.

The concept of an infinite well barrier means physically that the trapped particle has much smaller energy than the size of the well barrier.

To apply the result to a quantum well, we consider an electron moving in a nanostructure (with quantum size effects) confining this electron in a region L. In particular, in semiconductor materials we observe this quantum confinement by absorption and emission of light, that is, by its optical properties.

What is the dimension of that system for an electron moving at T temperature?. The electron moves only in one direction. Using the energy equipartition principle, it has energy of the order of $1/2(kT)$, where k is the Boltzman constant [8]; thus, using the last equation, L is:

$$L = \frac{1}{2}\frac{h}{\sqrt{mkT}} \quad (7)$$

In a semiconductor, the electron is not free, but bounded to the periodic potential of the crystal it is possible to represent that movement in a simple form; it's inertia to move is simulated with a "different mass" moving with an equivalent mass m* in the semiconductor. This statement is rigorously demonstrated in solid state textbooks [9]; a typical value is m* = 0.1m_o. At room temperature kT = 1/40 electron Volts (eV.). With these values we find that we must have L approximately 10 nanometers; this is the origin of the nanostructure word that is a structure nanometers large. Thus a “thin semiconductor layer of thickness of 1 micrometer is not thin for purposes of confinement. It is in fact a crystal which would not exhibit any quantum size effects. To observe quantum size effects, we require thinner layers.

The very small crystal dimensions required to observe quantum confinement in semiconductors are calculated in laboratory, using optical spectroscopy, observing the light absorption due to electron transition from the ground state to an excited state. The theoretical knowledge of the excited state is possible using quantum mechanics methods [10].

The ground state of a 10 nm GaAs quantum well using the energy value for the infinite well has a value of 57 meV. [11]; a more realistic value according to the experiment is 32 meV, which is obtained using a finite potential well. We have this result only using quantum mechanics methods [12].

Although in this problem, the infinite well model overestimates the confinement energies, it is a useful starting point for the discussion of the physics because of its simplicity. For more exactly values it is necessary to use the Schroedinger equation of Quantum Mechanics, see [13].

It is possible to make a nanostructure system with semiconductor doped glasses, such as CdS, ZnS, if they are introduced into the glass during the melting process, microcrystals are formed within the glass matrix. The dimensions of the microcrystals depend on the way the glass is produced. With careful preparation is possible to make nanocrystals with good size and uniformity [11].

3. Conclusion

The expression for the product of position and momentum obtained using the Bohr-Heisenberg microscope is a previous resemblance of the formal uncertainty principle of Quantum Mechanics, and in agreement with it, and could be useful for introduce the first time in elementary modern physics courses to the basic idea of the uncertainty principle and knowing the infinite potential well and the exact value of the ground state of energy. Besides the application introduced here have an idea of modern systems handled in solid state physics such as a quantum well.

The treatment of basic problems in Quantum Mechanics, using simple arguments gives to undergraduate students, in first courses of Modern Physics in Engineering, Chemistry and Physics Programs, the opportunity of obtain a general knowledge of them and a first idea of novel systems handled currenty in solid state physics.

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