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Knot numbers used as *labels* for identifying subject matter of a *khipu*

Identificación del tema del contenido de un *quipu* considerando a los valores numéricos de los nudos como *etiquetas*

Alberto Saez-Rodriguez[1]

Abstract

This investigation presents a new way to look at the numerical *khipu*, a knotted-string recording device from Pachacamac (Peru), and the types of information it contains. In addition to celestial coordinates, *khipu* knots apparently pertain to an early form of double-entry accounting. This study hypothesizes that the *khipu* sample has the recording capacity needed to register double-entry-like accounts. After the identification of its subject matter, the *khipu* sample was studied in an attempt to ascertain whether the knot values could represent instructions from the Inca state administration to a local accounting center. The results indicate that the numerical information in the pairing quadrants (determined by the distribution of S- and Z-knots) should be read from top to bottom along the full length of the string and can then provide certain complementary details regarding the projected corn stocks of the Inca state.

Keywords: Double-entry accounts, corn stocks, Pairing quadrants; S- and Z-knots; Subject matter of a *khipu*.

Resumen

Este estudio presenta un enfoque para extraer toda la información posible de una muestra de *quipu* numérico (sistema de cuerdas anudadas) de Pachacamac (Perú) ya que, además de conformar un sistema de coordenadas celestiales, podría contener, también, un sistema primitivo de contabilidad por partida doble. En el presente trabajo se hipotetiza que la muestra de *quipu* posee la capacidad necesaria para poder registrar dobles anotaciones contables. Luego de identificar el tema central del *quipu*, la muestra fue estudiada con el fin de intentar determinar si dicho *quipu* podría contener instrucciones contables provenientes de la Administración estatal inca destinada a un centro de contabilidad local. Los resultados indican que la información numérica contenida en los pares de cuadrantes (determinados por una distribución de nudos en S y en Z) debe ser leída desde arriba hacia abajo a todo lo largo de las tiras de nudos y que contienen informaciones numéricas complementarias referentes a reservas proyectadas de maíz.

Palabras clave: Contabilidad por partida doble; Provisiones de maíz; Pares de cuadrantes con nudos de S y de Z; Tema del contenido de un *quipu*.

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Introduction

Increase and further refine our knowledge and understanding of the patterns in the construction and distribution of *khipu* knots, the author re-examined the numerical *khipu* sample VA 42527. Beginning in the 1970s, Marcia Ascher (1986, 1991, 2002), both individually and with her husband Robert Ascher (1997), conducted invaluable research into the numeric significance of the *khipu* and developed a system of recording *khipu* details that is still widely used today among *khipu* researchers. In recent decades, there has been increasing research on the string-processing and manipulation procedures involved in the early phases of *khipu* construction. Most studies that have explored the analogy between the figure-eight knot (Figure 1) and the system of information registry encoded into the knotted *khipu* have addressed with Inca accounting (e.g., Asher, 1986, 1991, 2002; Urton 1998; Sarmiento de Gamboa, 1999: 41; Urton, 2003; Salomon, 2004).

Literature review of the *khipu* "accounting" system.

It is believed that the double-entry system of bookkeeping was introduced to Europe in the early sixteenth century by Luca Pacioli, the monk who tutored Leonardo da Vinci in mathematics. This system has been in use in most of the developed world for over 500 years and appears likely to remain a popular method for recording accounting transactions.

According to the mid-seventeenth-century Jesuit chronicler Bernabé Cobo (1983 [1653]: 253–254), the Inca designated certain officials to perform accounting-related tasks. These officials were called *quipo camayos*, and the Incas had great confidence in them. Therefore, I asked two key questions:

> [Q1] What statistical accounting (quantitative) form might the conventional Inca recording units (the numerical values for quantities) have taken?
>
> [Q2] Could numerical *khipus* contain fully coded information regarding a grain inventory/accounting system?

The first question addresses the issue of whether the *khipu* recorded information on materials in the state warehouses (*collcas*) in a statistical accounting (quantitative) form. The second question is more novel because it raises fundamental questions pertaining to the purpose and manner of use of the *khipu*.

Materials and Methods

Sample description.

The *khipu* sample VA 42527 (Museum für Völkerkunde, Berlin) contains an arrangement of one figure-eight knot tied to a separate string (i.e., pendant string n° 21). Figure-eight knots on decimal *khipu* normally signify the numerical value of one (Locke, 1923: 31). Our sample (see Figures 2 and 4) includes 21 pendant strings without any subsidiary strings. The primary cord is brown, whereas the pendant strings are white (Radicati di Primeglio, [1951], 2006). Urton's four-part organization of knot directionality (Urton, 2003: 87) was studied using this *khipu* sample (Figure 4). However, before beginning, it may be helpful to the non-Andeanist reader if I describe the basic features of the *khipu*.

Structural properties.

The study of *khipu* structural properties has been advanced in recent years by the research of Gary Urton (1994, 2003). Urton's focus on structure has heightened awareness of the degree of variability and patterning in spin/ply direction (see Figure 2), attachment type, and knot-direction variation (see esp. Urton, 2003: 74–88). In particular, Urton suggested that the S-knots in the upper left quadrant may be read together with those in the right quadrant, rather than simply read down the full length of the string from top to bottom (see Table 1 and Figure 2). Exploring this line of reasoning, Urton (2003: 153) noted that the knot pattern displaying Z-type single, long, and figure-eight knots is roughly twice as common as the pattern displaying S-type knots in all three positions (Figures 1, 2 and 3).

Khipu knots and their numerical values.

The first point to make regarding the registration of numbers in the *khipu* concerns the types of knots used (see Figure 1). There are three knot types: figure-eight knots, indicating ones (i.e., single units); long knots, which signify the units from 2 to 9; and single knots, which, depending on their position on a string, may signify any one of the full decimal units (i.e., 10s, 100s, 1,000s, or 10,000s). According to Marcia Ascher y Robert Ascher (1997 [1981]), there is a distinction between knot values signifying numbers used as magnitudes and knot numbers used as labels. The former constituted the basis of recording and accounting in the statistical (quantitative) *khip*u. Linking the notion of 'number labels' with recorded Quechua/Inca ideas regarding the use of both numbers and labels, we can interpret the knots of the *khipu* sample as both numbers and labels. Thus, for instance, one can argue that the Aschers' notion of 'number labels' could be interpreted as the information related to the general subject matter of the *khipu* (see Figure 5).

The *Khipukamayuqs* (*khipu* makers).

The former Inca record keepers – known as *khipukamayuqs* (knot makers/keepers) – supplied Inca rulers with a colossal variety and quantity of information pertaining to censuses, accounting, tributes, ritual and calendrical organization, genealogies, and other matters (Nordenskiöld, 1925; Asher, 1986, 1991, 2002; Urton, 2003: 3). Although the *khipukamayuqs* (the *khipu* makers) could "read" the *khipu* by running their fingers along the strings, no way currently exists to read the *khipu* that have survived. Other relevant factors could be revealed by the allocation of an appropriate organizing principle to the techniques for interpreting accounting values registered in the *khipu*.

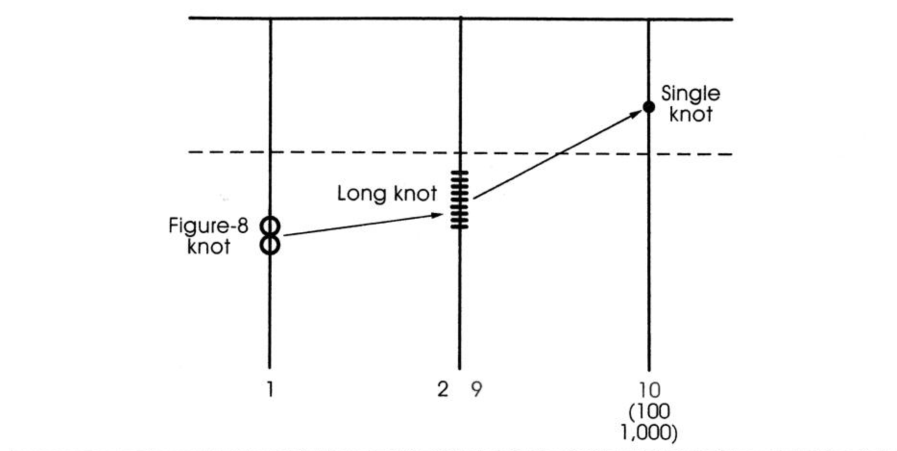

Figure 1. *Khipu* knot signs and their numerical values (Adapted from Urton, 2003)

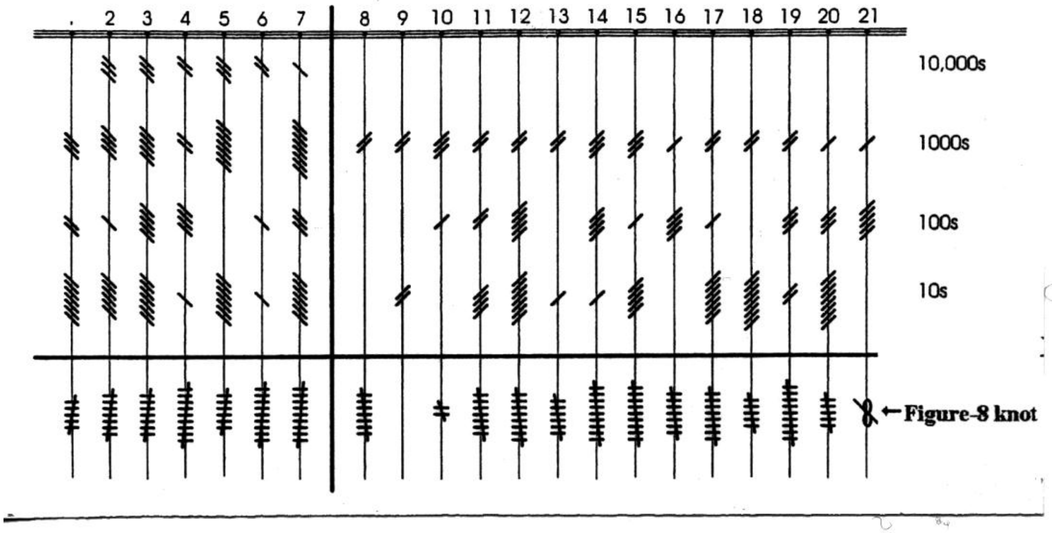

Figure 2. The four-part organization of knot directionality on the *khipu* sample (see Fig. 4) and the distribution of S-knots (1 to 7) and Z-knots (8 to 21). Adapted from Urton (2003, p. 87) with added elements.

Figure. 3. Long knots of types S (left) and Z (right). (Source: *Khipu* Database Project at Harvard University, 2009).

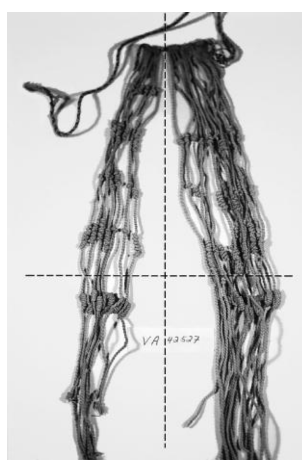

Figure 4. *Khipu* sample from Pachacamac (VA 42527, Museum für Völkerkunde, Berlin). Source: G. Urton (2003).

Procedure

Identifying the general subject matter of the *khipu* sample.

What was the *khipu* sample really counting? In the absence of a class indicator for the *khipu* sample, we do not know what objects were counted upon it. I therefore suggest that the *khipu* record keepers might have invented a configuration of string structures (e.g., the distribution of S- and Z-knots) to indicate the administrative class of the *khipu* in question, as well as its general subject matter and the magnitude of the units it recorded.

There was a close connection between the Pleiades (known as *Collcacapac* in Quechua) and the provision of food supplies. According to Randall (1987) and Laurencich-Minelli & Magli (2009: 118–119), the seven stars of the Pleiades were considered to be a "granary" (*qolqa*), and regular observations of these stars were conducted to help determine the times for planting and harvesting crops. This idea is also supported by Salomon & Urioste (1991), who found that the Huarochiri manuscript described the considerable effort made by the Inca in the observation of the Pleiades at dawn and stated that their brightness was used to forecast harvests. Brightness data would have given the Inca all of the information they needed to use from these stars for agricultural purposes.

In the research of Saez-Rodriguez (2012), a simple linear regression analysis was performed to translate the *khipu* sample's values (see Table 5) into a realistic-looking representation. The general graphical representation of the information in the pairing quadrants (as determined by the distribution of S- and Z-knots) can be used to gain an overall view of the subject matter of the *khipu*, i.e., to figure out what the *khipu* sample was really counting. In this analysis, it is noteworthy that the observed distribution of the long-knots' values (valuing each knot between 2 and 9 in accordance with its number of turns) along two perpendicular axes (x and y) corresponds to the brightest stars in the Pleiades cluster if one constructs a linear relationship using S-knots for *x*-values and Z-knots for *y*-values (see Figure 5).

In addition, the appearance of these stars coincided with the sailing season in antiquity; sailors were well advised to set sail only when the Pleiades were visible at night, lest they meet with misfortune. Thus, more concretely, my re-study of the *khipu* sample previously investigated by Nordenskiöld (1925) and Urton (2003) involved using the additional pieces of information identified through the linear regression analysis to produce a graphical representation of the organization of the stars of the Pleiades. In particular, the graphical representation of the Pleiades shown in Figure 5 appears to suggest that it corresponds to the general subject matter of this particular sample, i.e., maize (see Paragraph 2.1.2 regarding knot values).

Forms of Information.

I argue, as emphasized by Urton (2003: 78), that the idea described above would be a significant misinterpretation of the importance of these construction features and their variations within the *khipu* coding system, for three different reasons. Although corn production was extensive and state-organized in areas such as the Cochabamba Valley (in the Late Bronze Age), if traditional interpretations of the upper left quadrant of the *khipu* (see Urton, 2003: 87) are used to represent the enormous stocks of corn or projected corn stocks (i.e., carryover stock) accumulated by the Inca, the resulting estimates nonetheless appear quite excessive (i.e., 3,275; 34,167; 35,577; 22,419; 37,076; 21,109; 18,379), particularly after realizing that the values are expressed in *raki* and that each *raki* is 30 kg. I thus divided Urton's values (2003: 87) by 10. Table 1 illustrates the resulting values according to a system that mathematicians call "base 10." I did so because I believe that the "ones" position (i.e., the long knots) represents certain contextual information contained in the *khipu*; thus, the presumed "tens" actually represent "ones," the next digit represents the number of "tens", and this pattern persists up the entire length of the strings, resulting in the following values: 327/5; 3,416/7; 3,557/7; 2,241/9; 3,707/6; 2,110/9; 1,837/9. This interpretation would imply that the Inca could have "inscribed" (tied, dyed, and twisted) basic information on the *khipu* for later retrieval by trained *khipukamayuqs*. Assumptions that the upper left quadrant (Figure 2) could represent any of a range of products, such as bales of cotton (*utkhu*), bales of wool (*millma*), or a small cereal such as *quinoa* (pronounced keen-wa), still produce presumed figures that appear quite excessive, particularly when accounting for the weights and measures used by the Incas such as *poqchay* (11,502 kg each), *phoqcha* (60 kg each), *raki* (30 kg each), *ch'eqta* (1/2 *raki*), or *tupuna* (also known as *armu* or *armut*, and equal to 60 kg).

Arranging all pendant strings of the upper right quadrant in pairs.

If *khipu* can be deciphered, the analysis of the previous subsection is the type of approach that will accomplish that task (Urton, 2003). In undertaking this critique, I adopted a novel approach that is similar to that used by Saez-Rodriguez (2012). As stated above, although scribes (*khipukamayuqs*, the *khipu* makers) could "read" the *khipu* by running their fingers along the *khipu* strings, no method currently exists to read the *khipus* that have survived.

Corn was the most important crop in the Inca Empire. It was a major resource for nutrition, the basis for the corn beer called chichi that was consumed during all rituals, and a staple food crop for the peasant communities. Corn was a major tribute item provided to the state. Grown in state-controlled fields with the use of corvée labor (*mit'a*), corn was stored in massive granaries (storehouses for threshed grain or animal feed) established near the regional administrative centers of the Inca (Bauer, 1996). I therefore assumed that the records on our *khipu* represented the different varieties of corn during a given period of Inca culture.

The justifications for my assumptions.

In this particular sample, I also observed that half of the values on the 14 pendant strings (nos. 8 to 14 in the upper right quadrant) were higher than the remaining values (nos. 15 to 21). As Table 1 and Figure 2 show, we can thus arrange pairs of pendant strings (A and B) such that pendant string B represents a smaller (or equivalent) quantity than its partner (pendant string A). Pendant string A should, therefore, indicate values greater than or equal to those in pendant string B ($A \geq B$). In accordance with this reasoning, I found that the sum total of the numeric values contained within strings 15B, 16B, 17B and 18B (315 + 140 + 217 + 209 = 881) is a multiple of 29.5 (see Table 2), which makes sense because a synodic months (lunations) last approximately 29.53 days (see Nordenskiöld, 1925). According to this principle, if reading from the top to the bottom, we might interpret pendant string n° 1 as representing the quantities 3/2/7/5, or 327.5. Similarly, pendant string n° 2 would represent 3,416.7, and so forth, as illustrated in Table 1 (see Urton, 1994 for more details). Therefore, I agree with Urton (2003: 88) that we need to revise our approach to the study of the quantitative values recorded on the *khipu*.

These results indicate the existence of a preferential pairing assortment of strings. More careful observation revealed the more efficient logic apparently used by *khipucamayuqs* for arranging pendant strings in pairs, namely, pairing the first pendant string with the last, the second with the next to last, and so forth. Moreover, I made an attempt to examine the tributary system imposed by the Inca regime on the present *khipu* sample. As Table 2 shows, dividing the sum of the values of pendant strings 8A and 9A (200 + 202 = 402) by 27.25 (a time that corresponds to the length of the sidereal month, which is just over 2 days shorter than the lunar month of 29.5 days). Because this calendar was based on the moon, it had only 327 days, which corresponds to twelve months of 27.25 days each. It therefore appears that the tribute was assessed in the form of public labor, with the local *ayllus* ('local community') obligated to work in the construction of the *Qhapaq ñan* (the main 'north–south royal road' ranging for 6,000 km) for 14.75 months. Finally, dividing the sum of the numerical values of the pendant strings 19B, 20B and 21B (232 + 139 + 151 = 522) by 41 sets of 7.97 days (i.e., ≡ 41 eight-day weeks, totaling 327 days), we obtain 12.7 months, which corresponds to a twelve-month lunar calendar. Observations of the constellation Pleiades in the sky corrected the discrepancy in a 327-day year (see Figure 2).

Results

Results for string pairings in both upper left/right quadrants

Once our graphical representation (i.e., the Pleiades star cluster) has identified the *khipu* subject matter, linking the *khipu* sample to the Pleiades (with maize), we can gain a clearer understanding of the upper left quadrant (ply direction "S"). Projected maize stocks (i.e., carryover stock) were stored by the Inca for times of war or famine. Table 2 illustrates the information issued from the state administration to a local accounting center regarding the projected usage of the produced corn. It is widely believed that the Inca *khipus* contain calendrical notations, but no satisfactory demonstration of this notion has been possible. As Table 3 shows, if we divide the sum of the remaining balances (64 + 78 + 16 + 40 + 61 + 26 = 285) by 10 we obtain 28.5, which is very close to the time between lunations: ≈ 29 day lunar cycle. The Inca grain storage system for times of war or famine, which was based on a 10 month cycle (see Table 2), suggests the further possibility of a count by "tens" that could result in a formal year of 10 months. Although at present, corn storage lengths range from three to 24 months, at the time of the Inca Empire maize was stored for 10 months (10 synodic months, 10 x 29½ = 295 days), indicating that corn was commonly consumed prior to or by the time the new season's maize was ready for harvesting (see Tables 1 & 2). Therefore, months are divided into weeks of 10 days (NC, 260), as ten is the basis number for any Inca system of land organization.

I observed that the numbers arranged in calendrically significant patterns were used for agricultural purposes in the "farm account books" kept by the *khipukamayuq* (accountant or warehouse keeper) to facilitate the closing of his accounting books (see Tables 1, 2 and 3). It served, perhaps, as a basis for the calculation of the farmers' tribute (*ñan*) paid to the Inca emperors in the form of production. Although the meaning of *ñan* in Quechua is "road," the Inca people rendered abundant tribute in the form of production (and sometimes in the form of a labor tax) to the Inca emperors in the construction of the *Qhapaq ñan* (the main north–south royal road), including highland and coastal routes, according to the Inca tribute system. Although it is a highly speculative suggestion, I have long wondered if long S-knots could have been 'read' and given logographic values, e.g., words related to maize (see Table 3).

An analysis of the projected maize stocks on this *khipu* sample (Table 2) demonstrates that calculations can be performed using the totals in Table 2 (332 + 3,423 + 3,564 + 2,250 + 3,713 + 2,110 + 1,846 = 17,147), and the figures from the string pairs of Z-knots (8A + 9A + 10A + 11A + 12A + 13A + 14A = 1,774) – (21B + 20B + 19B + 18B + 17B + 16B +15B = 1,444) produce a difference of 1,774 – 1,444 = 330 when we subtract one total from the other.

According to Tables 1, 2 and 3, the Stocks to Use Ratio indicates the level of carryover stock for corn as a percentage of the total use. The mathematical formula for this relationship is as follows:

$$\frac{\text{Beginning Stock} + \text{Total Production} - \text{Total Use}}{\text{Total Use}} = \text{Stocks to Use Ratio}$$

This ratio can then be used to indicate whether current and projected corn stock levels are critical or plentiful. The ratio can also be used to indicate how many days of supply were available to the Inca under current usage patterns:

$$\frac{17147 + 1774 - 1444}{1444} = 12.10\%$$

The Inca had a lunar calendar, which had a 327 day year. Thus, as 327 x 12.10% = 39.56 days of available corn supply, we can speculate that a 12.10% stocks to use ratio for corn indicates that there were ≈ 40 days of supply of corn in the Inca reserves. The total production represents the total corn produced in a given year, whereas the total usage is the sum of all of the end uses by which the stock of corn was consumed. This usage would include human consumption, seed, waste, dockage and feed consumption (Garcilaso De La Vega, 1966 [1609-1617]; Murra, 1980).

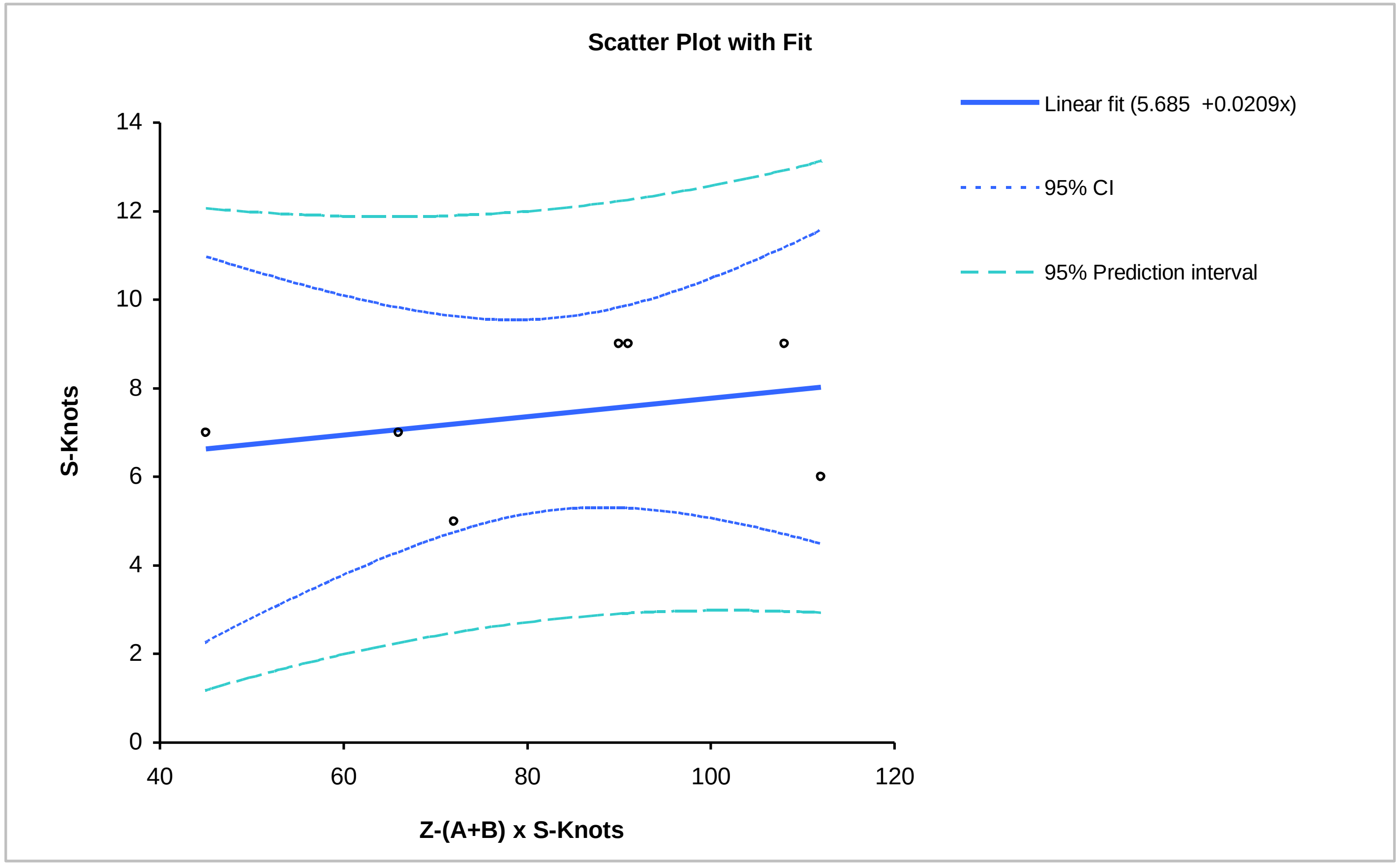

Figure 5. The locations of the brightest stars in the Pleiades cluster on the perpendicular axes of the standard *x*, *y*-plane.

Note: A set of 7 points (in black) indicates the positions of the 5 brightest stars in the Pleiades cluster, compared with the simple linear regression line (blue). When written as ordered pairs (*x*, *y*) in accordance with Table 2, the coordinates of the stars are as follows: Atlas (45, 7), Alcyone (66, 7), Merope (72, 5), [Maia (90, 9), Taygeta (91, 9)], Electra (108, 9), and Celaeno (112, 6). Adapted from Saez-Rodriguez (2012).

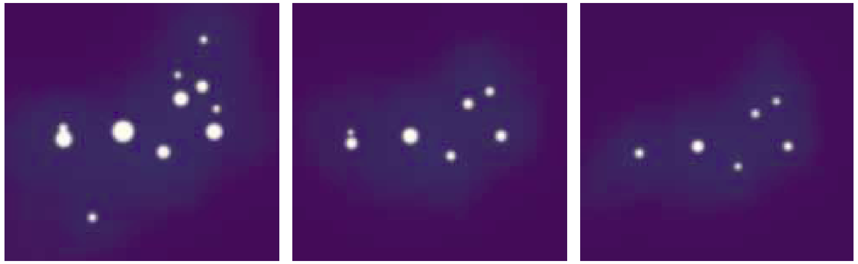

Figure 6. A simulated view of the Pleiades as they might appear during a normal year, when high cirrus clouds do not obscure the night sky; eleven stars are visible (left image). Viewing the cluster during an *El niño* year, when high cirrus clouds are more abundant, would reveal fewer stars (right image). Conditions between these extremes would allow an intermediate number of stars to be observed (middle image). Adapted from Orlove et al. (2002).

Table 1. The four-part organization of knot directionality of the *khipu* sample.

S-knots								Z-knots														
Granary stocks of corn*								Corn production in a given period							Projected usage**							
1	2	3	4	5	6	7		**8**	**9**	**10**	**11**	**12**	**13**	**14**	15	16	17	18	19	20	21	Decimal units
	30,000	30,000	20,000	30,000	20,000	10,000																10,000s
3,000	4,000	5,000	2,000	7,000		8,000		2,000	2,000	3,000	2,000	2,000	2,000	3,000	3,000	1,000	2,000	2,000	2,000	1,000	1,000	1,000s
200	100	500	400		100	300				100	200	500		400	100	400	100		300	300	500	100s
70	60	70	10	70	10	70			20		40	70	10	10	50		70	80	20	80		10s
3,270	34,160	35,570	22,410	37,070	20,110	18,370		2,000	2,020	3,100	2,240	2,570	2,010	3,410	3,150	1,400	2,170	2,080	2,320	1,380	1,500	Totals
327	**3,416**	**3,557**	**2,241**	**3,707**	**2,011**	**1,837**		**200**	**202**	**310**	**224**	**257**	**201**	**341**	315	140	217	208	232	138	150	Divided by 10
5	7	7	9	6	9	9		7	0	2	7	8	6	9	9	7	8	5	9	5	(1)	L-knots
332	**3423**	**3564**	**2250**	**3713**	**2020**	**1846**		**207**	**202**	**312**	**231**	**265**	**207**	**350**	324	147	225	213	241	143	151	Total

Note: The pairing quadrants were determined by observing the distribution of S- and Z-knots.

*Projected maize stocks (1-year projection period corresponding to the Inca lunar calendar of 328 days, i.e., a sidereal-lunar year).

**The rest was stored for times of war or famine.

As for a numerical *khipu* sample, we will see that the *khipukamayuqs* significantly modified and expanded their coding units to adapt this component of the statistical *khipu* information system from a real star map, although the general concept originated from within an overall statistical framework.

Table 2. The four-part organization of knot directionality of the *khipu* sample. The harvesting and storage of maize according to the lunar calendar.

S-knots | Z-knots

Granary stocks of corn*								Corn production in a given period							Projected usage**							
1	2	3	4	5	6	7		**8 Tribute**	**9**	**10**	**11**	**12**	**13**	**14**	15	16	17	18	19	20	21	Decimal units
◐								◐														
1-year								14.96 months														
	30,000	30,000	20,000	30,000	20,000	10,000																10,000s
3,000	4,000	5,000	2,000	7,000		8,000		2,000	2,000	3,000	2,000	2,000	2,000	3,000	3,000	1,000	2,000	2,000	2,000	1,000	1,000	1,000s
200	100	500	400		100	300				100	200	500		400	100	400	100		300	300	500	100s
70	60	70	10	70	10	70			20		40	70	10	10	50		70	80	20	80		10s
3,270	34,160	35,570	22,410	37,070	20,110	18,370		2,000	2,020	3,100	2,240	2,570	2,010	3,410	3,150	1,400	2,170	2,080	2,320	1,380	1,500	Totals
327	**3,416**	**3,557**	**2,241**	**3,707**	**2,011**	**1,837**		**200**	**202**	**310**	**224**	**257**	**201**	**341**	315	140	217	208	232	138	150	Divided by 10
5	7	7	9	6	9	9		7	0	2	7	8	6	9	9	7	8	5	9	5	(1)	L-knots
332	**3423**	**3564**	**2250**	**3713**	**2020**	**1846**		**207**	**202**	**312**	**231**	**265**	**207**	**350**	324	147	225	213	241	143	151	Total
17,148 ÷ 52 = 329.76 (13 months x 4 weeks = 52 weeks)* **								**409÷27.33=14.96**							**1,415**							

Note: The pairing quadrants were determined by observing the distribution of S- and Z-knots.
*Projected maize stocks (1-year projection period corresponding to the Inca lunar calendar of 327 days).
**The rest was stored for times of war or famine.
***The sum total of the numeric values contained within S-tied long knots: 5+7+7+9+6+9+9=52

Table 3. Total corn use projection. The target words in Quechua are italicized here.

Pairs of pendant strings n°	Number of turns (1 to 9)	Words in Quechua	In English	Total corn use projection	Remaining balance [g]	Sum of balances
8A	7	*aymuray killa*	Harvest moon	200		◐
21B	1	*ñan* [a]	Accounting of the labor tribute (whether paid, projected, or levied).	151	(50)	0
9A	[c]		[c]	202 [e]		
20B	5	*allpa wata* [d]	farm tenancy	138 [f]	64	64
10A	2	*qolqa*	corn silage bunker[b]	310		
19B	9	*Taqechakuy Machu Qolqa*	corn silage in Machu Qolqa	232	78	78
11A	7	*qachasqa ch'ila sara*	dried hard corn	224		
18B	5	*sara apita*	corn starch	208	16	16
12A	8	*chaminco opuka sara*	red corn	257		
17B	8	*paraqay sara chullu*	white corn to steep	217	40	40
13A	6	*paraqay sara*	white corn	201		
16B	7	*sara muhu wisiy*	thresh corn	140	61	61
14A	9	*shataychika kulli sara*	surplus purple corn	341		
15B	9	*chhanqayachiy sarata*	corn cob granulate	315	26	26
						285÷10=28.5*

Note: (a) The tribute rendered to the Inca Emperors in the construction of the *Qhapaq ñan* (e.g., the main north–south royal road).
(b) The quantity of corn transferred or conveyed to another granary.
(c) This string does not show any referral data at the "ones" position.
(d) The form of lease arrangement.
(e) Total farms (the number of *chakra*);
(f) The ratio of tenants to total farms.
(g) The remaining balance: the corn that was stored for times of war or famine.
*The time of each lunar cycle ≈ 29 days.

Discussion

What assumptions are being made about the nature of Inca recordkeeping in the *khipus*? What are the bases for these assumptions (e.g., why should we accept that the *khipu* VA 42527 records information about corn when, in reality, potatoes are a crop that is as significant, if not more so, in the Andes)? To answer the previous question with a question: How does one write a book report without reading the book?

As Tables 1 and 2 show, the first two pairs of strings (8A+21B; 9A+20B) are theoretically possible related to the tribute calculation (*ñan*), and the third pair of strings (10A + 19B) specifies a quantity of corn conveyed to another granary, whereas the remaining pairs (11A + 18B; 12A + 17B; 13A + 16B; 14A + 15B) address the corn use projections in accordance with instructions from the state administration. Tables 1 and 2 show the granary stocks of corn or the corn production in a given period and its projected usage, in accordance with the instructions issued from the state administration to a local accounting center. It is known that Incan farmers grew more food than was needed (Pärssinen, 1992). After part of the harvest was eaten, the rest was dried and stored for times of war or famine. As shown in Table 1, there were significant differences between quantities in the two quadrants. For example, the upper left quadrant indicates values greater than 30,000 units, numbers that are normally registered as complete counts (Urton, 2003: 91) and that are higher than those shown in the upper right quadrant (up to 3,000). This observation suggests that the quantities shown in the upper left quadrant are the granary stocks of corn (or projected corn stocks), whereas the quantities shown in the upper right quadrant refer to the corn production of only one year.

Ascher and Ascher (1997, p. 78) argue that the *khipukamayuq* "used cues from a shared informational model within Inca culture, and particularly those aspects of the model related to state affairs". The question, however, is where we locate a model for conceptualizing the organization and meaning of S/Z decision making in the Inca *khipu* that is consistent with the powerful principle of statistical organization employed in every other aspect of the construction of this sample.

Curiously, according to De la Calancha (1974: 205) and Garcilaso De La Vega (1966), a native speaker of Quechua, the *khipukamayuq* fit their values to the *khipu* structures (rather than the reverse mechanism); moreover, someone other than the "*khipu* makers" might have constructed the *khipu*, and it was the *khipukamayuq*'s job to find a means to fit the structure of values in question to the (created) *khipu*.

Conclusions

In this particular sample, the information in the upper right quadrant specifically supports previously promulgated notions regarding Inca accounting practices, and the information in the pairing quadrants (as determined by the distribution of S- and Z-knots) should be read from top to bottom along the full length of the string, including the details contained in the long knots at the figure-eight knot. Finally, Tables 3 and 4 depict an accounting and bookkeeping system for an agricultural cooperative consisting of a large number of small cultivated fields (*chakra*) that were organized according to their own particular needs. This *khipu* accurately reflects known realities, i.e., the production, planning and tribute paid to the Inca rulers (Inca emperors). Contextual information contained in the long knot (9 turns) on pendant string 19B indicates that a fixed quantity of corn (i.e., the magnitude of units recorded: 2,320) was assigned to be stored at *Machu Qolqa* (the Warehouse of the Inca). Apparently, the grain warehouse keeper kept position records for each type of grain, ensuring that both the grain warehouse keeper and the Inca state administration could easily determine the total amount of grain held by the warehouse keeper. The following question should now be addressed: how could the Inca 'pseudo' longitudes and latitudes have been central to *khipu* coding and decoding when none of the salient features of such a system was even mentioned by the chroniclers? I believe strongly that the 'pseudo' spatial reference system (also called the coordinate system) was a particularly complex system at the time of the Spanish Conquest, which began in 1532 and that it was, therefore, very difficult for historians to describe and explain the basic features of this remarkable system.

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