Chahine, I. C., & Naresh, N. (2013). Mapping the Cognitive Competencies of Street Vendors and Bus Conductors: A Cross-Cultural Study of Workplace Mathematics. *Revista Latinoamericana de Etnomatemática*, 6(3), 7-35.

Artículo recibido el 30 de mayo de 2013; aceptado para publicación el 15 de septiembre de 2013

Mapping the Cognitive Competencies of Street Vendors and Bus Conductors: A Cross-Cultural Study of Workplace Mathematics

Mapeo de las competencias cognitivas de los vendedores ambulantes y los conductores de autobuses: Un estudio transcultural del lugar de trabajo de Matemáticas

Iman C. Chahine[1]
Nirmala Naresh[2]

Abstract

This paper explores the mathematical ideas that emerge across two workplace settings, namely street vending and bus conducting. The purpose of this study was to delineate a trajectory describing potential mathematical structures underlying bus conducting and street vending activities and to extract a conceptual model that could explain the nature of the practitioners' mathematical knowledge and its connection to formal mathematics. We conducted a meta-analysis of the problem solving behavior and narratives of street vendors and bus conductors in two geographic sites in Beirut, Lebanon and Chennai, India. Principled by Vergnaud's theory of conceptual fields, the researchers examined heuristics-in-action that transpired as a result of practitioners' engagement in their respective work situations.

Keywords: workplace mathematics; ethnomathematics; heuristics; problem-solving; conceptual fields.

Resumen

Este artículo explora las ideas matemáticas que surgen a través de dos lugares de trabajo, la venta ambulante y la conducción de autobús. El propósito de este estudio fue definir una trayectoria que describa posibles estructuras matemáticas subyacentes en la conducción de autobuses y actividades de venta ambulante para extraer un modelo conceptual que podría explicar la naturaleza del conocimiento matemático de los practicantes y su conexión con las matemáticas formales. Hemos llevado a cabo un meta-análisis de la conducta y narraciones de los vendedores ambulantes y conductores en dos sitios geográficos en Beirut, Líbano and Chenai, India. De los principios de la teoría de los campos conceptuales de Vergnaud, los investigadores examinaron la heurística-en-acción que se dieron como resultado de la participación de los practicantes en sus respectivas situaciones de trabajo.

Palabras clave: matemáticas del lugar de trabajo; Etnomatemática; heurísticas; resolución de problemas; campos conceptuales.

[1] Department of Middle-Secondary Education. Assistant Professor of Mathematics Education. Georgia State University. Email: ichahine@gsu.edu
[2] Department of Mathematics. Assistant Professor. Miami University. Email: nareshn2@miamioh.edu

INTRODUCTION

In the past two decades, researchers have increasingly emphasized the elicitative role of cultures as they affect thinking and problem solving. For many researchers, sociocultural settings not only determine how knowledge is acquired, but also how it is represented, organized and retained (Sanin & Szczerbicki, 2009). As such, studies on cognition have capitalized on the role of contexts and experience in shaping our cognitive competence. For example, Masten and Coatsworth (1995) viewed competencies as "a pattern of effective performance in the environment, evaluated from the perspective of development in ecological and cultural context" (p. 724). A heightened emphasis on the situated nature of thinking and its adaptability to variations in social and cultural contexts have prompted new paradigms that seemingly challenge the traditional views that focused on mental processes disconnected from the setting in which the processes are acquired. Central to the idea of situatedness of thinking is the role of the environment in shaping and guiding an individual's cognitive and epistemic actions (Kirsh & Maglio, 1994). As such, cognition becomes an extension of the body and the environment that provides a rich medium of information. Advocates of the "extended mind" thesis go even further to suggest that cognitive processes as well as mental states, such as beliefs, are not necessarily stored in our heads but can also be stored in external representations (Hochstein, 2008). These approaches atypically emphasize the notion that the brain and, consequently, the mind and its processing cognitive faculties are continuously developing as a result of everyday interactions with the physical land social world. According to these approaches, cognitive processes are not limited to the internal processing of symbols but are employed in various perceptual and motor modalities (Reichelt & Rossmanith, 2008).

The implications of the situated cognition paradigm for exploring workplace mathematics are paramount. Lakoff and Nùňez (2000) proposed a theory of embodied mathematics in consonance with the study of situated cognition. The authors further argued that "the only access that human beings have to any mathematics at all, either transcendent or otherwise, is through concepts in our minds that are shaped by our bodies and brains and realized physically in our neural systems" (p. 346).

For some time, educators who have studied learning that occurs in informal and formal settings have recognized that children do well in their daily life and indeed grow as successful citizens in spite of their poor performance in school mathematics (D'Ambrosio, 1992). For example, Saxe (1988) showed that Brazilian candy sellers with little or no schooling can develop as a result of being immersed in the selling experience several successful practices that differ from what is taught in schools. Increasingly, educators have found the cultural surroundings of children to be a factor affecting their achievement in school mathematics (Dawe, 1988), providing support to the hypothesis that cognitive competence, learning capabilities, and attitudes towards learning are closely related to cultural background (Wang et al., 2004). Outside the school environment, children and adults who immerse themselves in different work-related practices perform "mathematically" well and develop novel problem solving strategies prompted by instrumental cognition (Carraher et al., 1985; Saxe, 1991; Jurdak & Shahin, 1999, 2002; Naresh, 2008).

We concur with some contemporary mathematics education researchers who conceive of thinking (including reasoning and problem solving) as a sociocultural practice that is experience driven (e.g., Magajna & Monaghan, 2003; Masingila, 1994; Noss, Hoyles, & Pozzi, 2000). There is strong evidence in the literature on situated cognition that supports the hypothesis that by actively engaging in everyday activities, individuals gradually incorporate culturally constructed artefacts into their repertoire of thinking and further develop context- specific problem solving competencies (Wenger, 2000). Such evidence predictably challenges the conventional definition of what counts as mathematics by reinforcing the claim that mathematical activity can be seen as interwoven with everyday practice outside the academic formal settings.

Repeatedly, mathematics education researchers have questioned the mathematical ideas that are generated and used outside of learning institutions (Millroy, 1992). This is the mathematics that allows people with little or no schooling experience to practice crafts and trades, conduct business transactions and make their livings in a variety of ways. This mathematical activity has been called “informal” mathematics (Ginsburg, 1988) or “everyday” mathematics (Lave, 1988) or “ethnomathematics” (D’Ambrosio, 1992), or "street" mathematics (Nunes et al., 1993), or "frozen" mathematics (Gerdes, 1998), or even workplace mathematics (Bessot & Ridgeway, 2000; Hoyles & Noss, 2010, Zevenbergen, 2000). Several contributions to the literature on workplace mathematics can be grouped into 3 major classes of studies on work contexts that differ in its focus on the degree of school mathematics involved and the level of competency required to conduct the work activity. This body of literature includes (a) studies that describe cases where there are high level of school mathematics and high competency required, including those that examine the work of engineers (Standler, 2000); (b) studies that describe cases where there are high levels of school mathematics needed but low level of competency involved, such as the work of nurses (Hoyles, Wolf, Molyneux-Hodgeson, & Kent, 2002); and (c) studies that show low or minimal systematic transmission of school mathematics and where workers continuously modify and appropriate their thinking strategies to serve practical needs of the work activity (Nunes et al., 1993). Our study focused on this group of workers. This line of investigation has shown that it is one thing to learn mathematics in school and quite another to solve mathematics problems intertwined in everyday activities "Whether it is inventory taking at work or shopping or calculating calories in cooking, school mathematics does not play a very important role" (Nunes et al., 1993, p. 3).

By the same token, work in non-Western contexts showed that several groups of people who learn numeracy without schooling use their indigenous counting systems to solve arithmetic problems through counting, decomposition, and regrouping (Gay & Cole, 1967; Ginsburg, 1988). For example, Gay and Cole (1967) reported that the Kpelle people of Liberia used stones as support in solving arithmetic problems and could solve addition and subtraction problems using numbers up to 30 or 40 with accuracy. Mercier (2000) argued that while some of the computational strategies employed by such workers are relatively complex and multivariate, the mathematical competency developed is, to a large extent, integral to the tools used and not to the user.

Hence, the idea prevails that mathematics in these work settings has its own forms that are adaptations to the goals and conditions of the activities. Zevenbergen (2000) highlights the importance of workplace mathematics in uncovering the unique forms of mathematics that emerge in the job-related activities.

STUDY RATIONALE

We have been teachers of mathematics for over 20 years in a wide range of settings and in many Western and non-Western countries, from K-12 schools to research universities and graduate schools of education. In our studying, teaching and research, we have encountered many implicit and explicit questions about intellectual competence and its role in defining and shaping the identity of people in the classroom, the workplace, and the society as whole. Our experiences as international scholars growing up in diverse social milieus and teaching in the United States juxtaposed with our research experiences in transcultural work-related contexts prompted our interest in the level of analytic complexity in everyday practices that thrive in both of our cultures. This led us to ask the questions: What are the mental processes underlying an act of labor or service? What is the nature of the problem solving behavior of workers while immersed in everyday work practice? How is experienced knowledge represented and employed as part of the daily decision-making manners undertaken by workers?

Researching in the context of the workplace provided us with just the right context for furthering this agenda. It also afforded us the opportunity of making connections between what are seemingly two disparate worlds —the world of mathematics learning and the world of mathematics in work. This paper involves a meta-analysis of the problem solving behavior and narratives of street vendors and bus conductors based on the findings of two studies where we explored how mathematics was used in various work contexts in order to tease out its potential and limitations in terms of developing useful models of work practices. The main purpose of this study is to: 1) to delineate a trajectory describing potential mathematical structures underlying bus conducting and street vending activities; and 2) to extract a conceptual model that could explain the nature of the practitioners' mathematical knowledge and its connection to formal mathematics.

THEORETICAL FRAMEWORK

We adopt Vergnaud's theory of conceptual field as the general theoretical framework for analysis to categorize vendors and bus conductors' problem solving behavior and narratives in the work settings. Vergnaud's (1988) theory of conceptual fields is based on the idea that concepts always involve three facets: invariants, representations, and situations. Invariants refer to the mathematical properties or relations associated with the concept. Vergnaud contended that invariants are expressed through representations and that they are not the only factor affecting performance. For instance, the way in which concepts are formed might be an essential factor to consider. Also, concepts are always tied to situations which make them meaningful. More importantly, Vergnaud (2000) argued that the existence of these mathematical concepts does not necessarily mean that people are fully aware that they are behaving accordingly, but most often these concepts are only "implicit" in theorems or what he calls "theorems-in-action," Vergnaud (1988) defined theorems-in-action as those "... mathematical relationships that are taken into account by students when they choose an operation or a sequence of operations to solve a problem" (p.144). Vergnaud (2000) further differentiated between what he called *concepts-in-action* and *theorems-in-action*. He described a theorem-in-action as "a sentence that is held to be true in action," whereas "a concept-in-action is to be held relevant or irrelevant" (p.22). For example, he explained that the concept of a scalar ratio in a missing value proportion problem is a concept-in-action, whereas the isomorphic property of the linear function, $f(a*x) = a*f(x)$, is a theorem-in-action.

The theory of conceptual fields brings us to the idea that to understand how mathematical concepts are acquired it is necessary to analyze the situations through which these concepts were made meaningful and useful in the context in which they are invoked. Vergnaud's model has provided not only guidelines for coding vendors' and bus conductors' problem solving behaviors, but also an understanding of the underlying properties and relations implicit in these behaviors. Pursuing this model, the vendors' and bus conductors' problem solving behaviors were coded into major categories, we called "heuristics-in-action," which were further analyzed into subcategories (i.e., strategies and substrategies). As coded categories began to emerge, we explored similarities and differences by comparing data across cultural settings and work activities. Heuristics-in-action that were extracted through the analysis of procedures and which emerged as a result of practitioners' engagement in their work situations are mechanisms that transform the learner's implicit mathematical knowledge into explicit mathematical theorems.

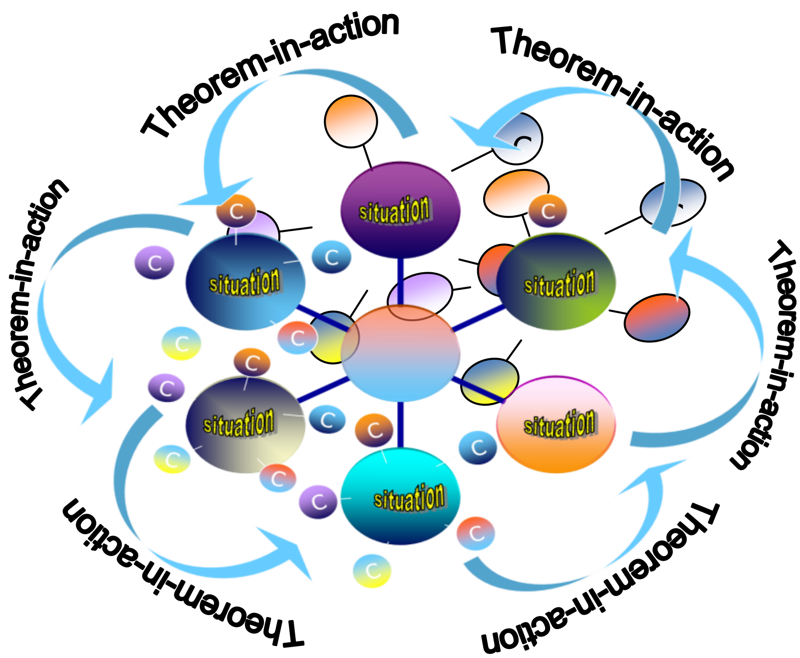

Figure 1. Authors' depiction of Vergnaud's Theory of Conceptual Fields.

THE STUDIES

The two studies we conducted involved the practices of street vendors in Beirut (Jurdak & Shahin, 1999) and bus conductors in India (Naresh, 2008). These studies not only contributed to the broader research fields of ethnomathematics and everyday cognition but also helped us gain insights into the workplace mathematical activities of the two groups. The overall goals of the case studies were to unravel, analyze, and describe the mathematical ideas and decisions employed by the participants to solve work-related mathematical tasks. While the two groups are different in three ways (age group [children vendors versus adult bus conductors], cultural context [Beirut vs. India], and incomparable mathematical entry requirements), both groups have three things in common:(a) They engage with mathematics in ‘error-critical’ activity (i.e., there is little or no room for error); (b) they spend long hours engaged in their work; and (c) competition is usually high, which prompts workers to examine and revise their decisions continuously and unexpectedly.

Our initial aims were exploratory: We wanted to examine cognition at work in order to define and describe practical mathematical knowledge that emerges in the context of work activities. Our approach therefore involved a number of methodological schemes (e.g., case studies, secondary data analysis of the case studies – case refers to the groups of bus conductors’ and street vendors) to extract those “mathematisable elements” in the participants’ behavior while immersed in their work practice. Our investigation was guided by a series of questions addressed for two main purposes: 1) to delineate a trajectory describing potential mathematical structures underlying bus conducting and street vending activities; and 2) to extract a conceptual model that could explain the nature of the practitioners’ mathematical knowledge and its connection to formal mathematics.

METHODS

Data collected from the authors’ research studies on street vendors in Beirut and bus conducting in Chennai comprised transcribed interviews, researchers’ introspection notes, problem solving narratives, and work sample artifacts. Two groups of practitioners were investigated in both studies. In the street vending context, participants included 10 male vendors randomly selected from two market settings in the southern suburbs of Beirut that were completely demolished during the Israeli invasion in 2006. Vendors in the sample varied in years of schooling (3-7 years), in age (10-16 years), and vending experience (1-8 years). Four of the vendors worked alone, while the other six helped their fathers or neighbors. Only three were totally responsible for purchasing the produce at wholesale market and pricing it for selling.

In the bus conducting context, five bus conductors were included. These bus conductors varied in their educational qualifications (2 had high school diplomas and 3 had Bachelor degrees) and years of experience (9-31 years).

Both groups worked in relatively densely populated neighborhoods in Beirut and Chennai and often came from a low socioeconomic background where family members, including children, normally work to support the family. In both work settings, the competition was usually high as the vendors and bus conductors would constantly be obliged to change their selling prices, revising, thereof, their profit and loss unexpectedly during the same day. Furthermore, both groups of practitioners spent long hours during in their jobs; vendors worked 6-7 days per week for 10 hours per day, and bus conductors worked up to 3 shifts for 14 hours per day. During the course of their daily work, both groups of practitioners engaged in transactions that required instantaneous decision-making processes involving mathematical procedures without the use of computational tools.

DATA ANALYSIS

Our general methodological approach was based on an iterative process of data collection, analysis and hypothesis generation in order to expose the underlying mathematical structures (i.e., for purpose 1) and to generate an explanatory model for both work practices (i.e., for purpose 2). Our methods comprised secondary data analysis (data collected from the larger studies), narrative inquiry of solution schemes and focused discussion (researchers as participants).

We employ secondary data analysis using data collected from our previous research with bus conductors in Chennai and street vendors in Beirut. The rationale behind using secondary data analysis was to reuse existing data collected from our previous enquiries in order to gain an in-depth knowledge on a specific topic: What is the nature of the mathematical ideas that transpires in the workplace? Moore (2006) argued that qualitative secondary data analysis can be understood not so much as the analysis of existing data but rather as involving a process of recontextualizing, and reconstructing data by “opening up a more productive notion of reuse and more possibilities of meaning-making from reusing data” (p. 21). Examining existing data from the two studies enabled data linkage that afforded powerful insights into the problem-solving behavior of practitioners while immersed in their respective work environments. Furthermore, revisiting data related to field observations, interviews and researchers’ notes in two work place contexts and two different countries allowed transparency within research as we continuously interrogated the quality of qualitative data in terms of coding and completeness. Secondary data analysis enabled us to learn from each other and to reflect on the respective strengths/weaknesses of pre-existing data sources.

Narrative inquiry of solutions schemes

Our attempts to gain meaningful insight into the "figured worlds" of street vendors and bus conductors while immersed in their workplace practices included analyses of the narratives that emerged during problem solving episodes in practitioners' daily work. Coulter and Smith (2009) argued that narrative inquiry studies "rely on stories as a way of knowing. Stories emerge as data are collected and then are framed and rendered through an analytical process that is artistic as well as rigorous" (p. 577). According to Bruner (1986), narrative thinking is one mode of human cognition and thus plays a crucial role in the way people make sense of their worlds. In a similar vein, Amsterdam and Bruner (2000) defined narratives as "mental models representing possible ways in which events in the human world can go" (p. 133). Through fine-grained analyses of the problem-solving narratives of practitioners at work, it was possible to extract important mathematical aspects of the practitioners' thinking processes in the workplace setting.

Focused discussions

To generate the development of ideas and thus extract significant assertions related to problem-solving behavior and narratives of practitioners, we frequently engaged in focused discussions during the course of the study. These discussions targeted specific mathematical frames that were captured as the practitioners are immersed in the context of street vending and bus conducting. Such discussions produced nuanced insights that would be less accessible without our intensive face-to-face purposeful interactions. As both of us come from two different cultural settings, the facility of holding and presenting diverse opinions with relative ease represented a considerable strength of the method and afforded an open space for negotiation and mediation and for establishing a tone of trust and provided group members with some image of each other. As we listened to each other's verbalizing and recollecting our field experiences memories, ideas, and experiences were stimulated and validated as we discovered a common language to describe our recollections and reveal shared understandings or common views. Lindlof and Taylor (2002) described the group effect where group members engage in “a kind of ‘chaining’ or ‘cascading’ effect; talk links to, or tumbles out of, the topics and expressions preceding it” (p. 182)

RESULTS

Our first approach towards analysis was to develop a scheme or a coding system in order to organize data collected from the two case studies. Developing a coding system for the problem-solving behavior and narratives of vendors and bus conductors involved careful readings of transcriptions (secondary data analysis) taken from practitioners’ written solutions as well as interviews, with particular attention to researchers’ comments. The second part involved extracting patterns of categories that emerged after reading, systematically searching and arranging the transcriptions and field notes, organizing them, and breaking them into manageable units. The third part involved scanning, through systematic content analysis, of the units of analysis in order to generate descriptions of the properties they contain. The latter two stages of data analysis required us to engage in focused discussions centered on the secondary data collected through the case studies. A qualitative analysis of the problem-solving behavior of vendors and bus conductors was established by comparing, contrasting, and synthesizing these properties across work settings namely, vending and bus conducting, and across cultures (i.e., Lebanon and India). We conducted three comparisons to decode and examine the problem-solving behaviors of street vendors and bus conductors (see Figure 2). Comparisons were carried out along three major dimensions: (a) representation systems; (b) heuristics-in-action; and (c) situations.

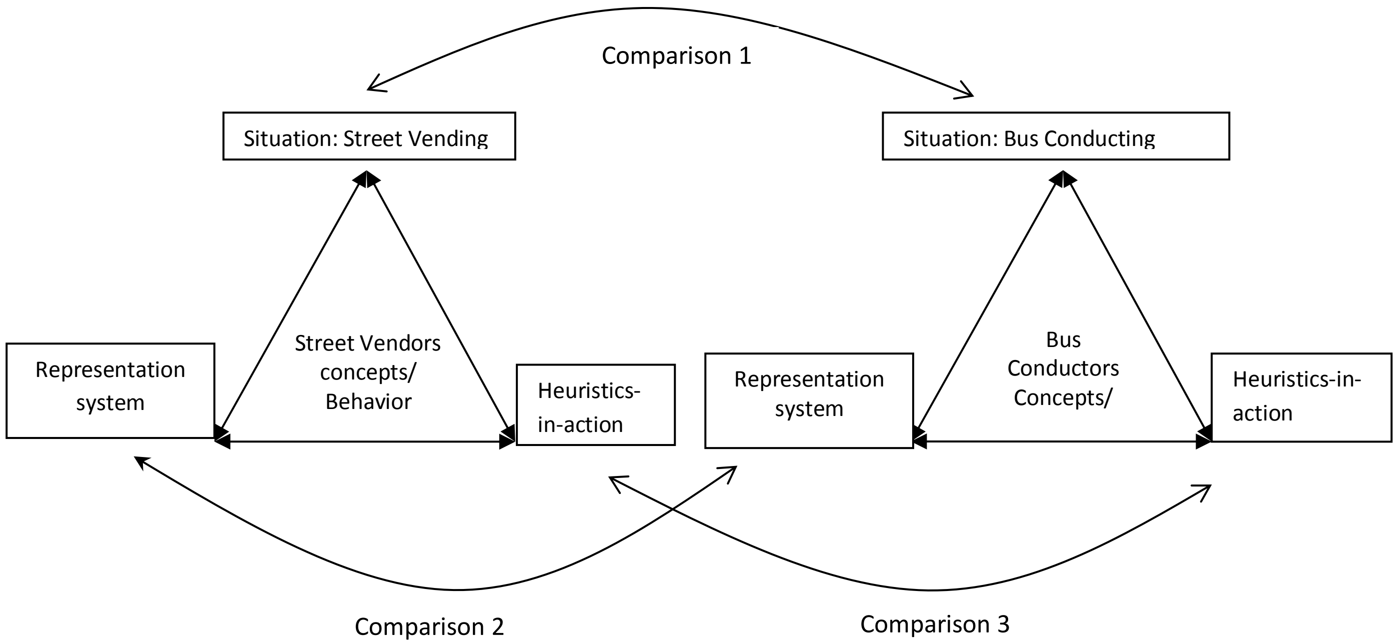

Figure 2. Analysis of street vendors and bus conductors' problem solving behaviors.

The first comparison is set between the two work situations (i.e., street vending in Beirut, Lebanon and bus conducting in Chennai, India). The second comparison involved examining the representation systems employed by both types of practitioners as they engaged in their daily work practice. The third comparison entailed a qualitative assessment of the mathematical elements underlying the computational and decision-making strategies that vendors and bus conductors attempt to resolve a work-related problem. Qualitative comparison of vendors and bus conductors' problem-solving behavior was also established. In addition, percentages of correct responses to problems were also computed in order to examine the effectiveness of the strategies employed by both groups of workers.

Comparison 1: Street vending and bus conducting situations

This dimension includes the comparison of the ways in which vending and bus conducting experiences and problem situations have affected the practitioners' mental frames. We argue that work situation provides its practitioners with meaningful, specialized ways of thinking and problem solving. As such, context-specific cognitive models, schemata, or mental frames are operationalized and become highly functional in work-related situations. As empirical findings have shown, cognitive domains arise in response to the demands of the physical and social environments in which individuals are immersed. For example, Australian aboriginal people of the desert regions develop extraordinary visual-spatial memory (Kearins, 1981); the unschooled young street vendors in Lebanon (Author, 1999) and in Brazil (Carraher, Carraher, & Schliemann, 1985; Ceci & Roazzi, 1994; Saxe, 1991) are capable of solving complicated math problems in their heads; Kpelle people of Liberia used stones as support in solving arithmetic problems (Gay & Cole, 1967). In each work situation, the physical and social challenges confronting practitioners demand different work-related skills, which become honed to different level of expertise. Millroy (1992) emphasized the importance of apprenticeship as a learning model: "Practice is valuable; the building of experience, with success and failure along the way, is the way in which learning occurs. Collaboration is important; for encouragement, for joint problem solving, for sharing new ideas, for teaching novices, for reaching new levels of development with the help of a more expert colleague" (p. 190). Consonant with this belief, we argue that work-related situations facilitate certain mental strategies and mind frames that shape practitioners' cognitive profiles. The skills and competencies that practitioners develop as a result of long immersion in their work situations provide insight into questions about how these workers identify, analyze evaluate, and solve problems that further facilitate their cognitive growth in important spheres of their daily life.

For the street vendors and the bus conductors, their work situations exposed these practitioners to vending experiences, which posed challenges and opportunities for purposeful problem solving. As a result, they invented many computational strategies for the purpose of achieving a transaction or a possible transaction. These vending-specific computational strategies were quick, active, spontaneous, and efficient in maximizing profit and minimizing loss. The natural work situation has shaped the vendors' ability to tackle a wide range of problems. As a result, the two groups systematically and smoothly built up their solutions using intuitive computational strategies and without losing track of the goal (i.e., maximizing profit) even if many numbers are involved. In other words, the two groups of workers kept the meaning of the problem in mind during problem solving. This understanding that the participants acquired in the work situation elicited a coherent problem-solving behavior that was attained through the following steps: (a) translating the problem from its real life context into an appropriate mathematical calculation problem, (b) performing the mathematical calculations, and (c) translating the result of this calculation back into the context of the problem to see whether it made sense.

Expertise in work situation capacitated the street vendors and bus conductors to develop computational strategies that facilitated their daily work activities. For example, as mentioned elsewhere (Author, 1999), we identified three basic types of computational strategies used to solve problems: (a) chunking for solving addition problems; (b) counting-up, for solving subtraction problems; and (c) repeated grouping, which was employed for multiplication. As an illustration of using decomposition, we introduce the following excerpt:

> Ali was selling garlic, 4000 L.L. /kg (L.L= Lebanese Lira; Kg= kilograms).
> Researcher: I would like to take 5½ kilos, how much do I owe you?
> Ali: 1 kilo for 4000 L.L. then 4 and 4 is 8, another 4 and 4 give (pause) 8 and 8 is 16, then 16 and 4 (pause, thinking) and ½ kilo for 2000 L.L., hence 22000 will be the cost of 5½ kilos of garlic.

Ali decomposed 5½ kg into the following sum: {[(1+1)+(1+1)]+1+1/2} kilos and accordingly, then computed the retail price through the same structural decomposition of the price, namely {[(4000 +4000) +(4000 +4000)]+ 4000 + 2000} L.L. Reasoning by establishing an isomorphism between the quantity of produce on one side and corresponding price on the other is a high-order cognitive mechanism that reduces cognitive load as the vendors are solving problems involving large numbers. This led us to conclude that work situation has motivated the vendors to unconsciously mathematize and conceptualize many ideas that are grounded in everyday work experiences.

In the case of bus conductors, we have documented the solution strategies the conductors employed to enact ticket (a token of travel) transactions (Naresh, 2008) (See Figure 3 for an example).

Starting and Ending Numbers on Ticket Bundles	Ticket Denomination.	Number of Tickets Sold	Researcher Identified Solution Strategy
34159 – 34177	12.00	77 – 59 = 18	To find 18 x 12, first find 20 x 12 as 240 (Reformulation) Compensate for the extra amount by subtraction 12 x 2; = 240 – 24 = 216 (Use of distributive property)

Figure 3. An example to illustrate conductor-employed mental computation strategies

Comparison 2: Street vendors and bus conductors' representation systems

The representation systems employed by vendors and bus conductors when writing their problem solutions in their work settings included different means of expressing ideas such as colloquial language, mathematical notations or signifiers, and pictorial representations. We considered three properties of representation systems employed by the practitioners in their work activity: formalistic; operative; and pragmatic (see Figure 4).

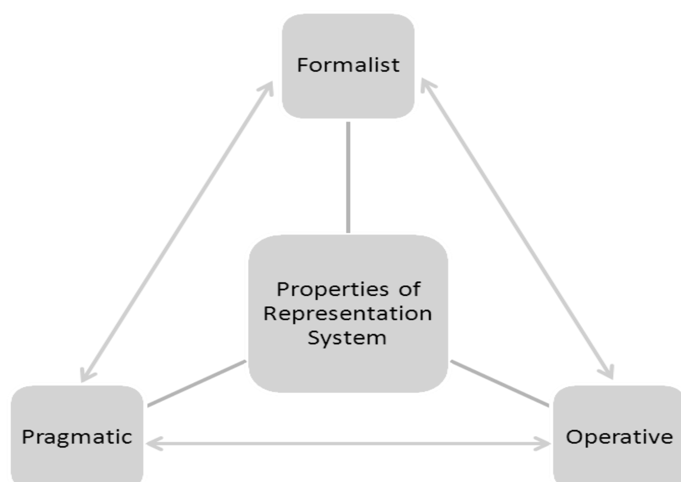

Figure 4. Properties of representation systems employed by street vendors and bus conductors.

We define formalistic property of a representation system as referring to the structure, style or mode of representing ideas including utterances, gestures or written schemes that practitioners employed to describe, express, and give meaning to immediate situations encountered in their work-related setting. A common representation used to solve problems is the verbal explanation of responses; vendors as well as bus conductors exclusively used mental, oral computational strategies invoked from memory. On the other hand, the operative property refers to the way practitioners actually perform the activity of problem solving using various representation modes or structures such as manipulating quantities by operating on symbols using mathematical properties and referring the solution back to the situation at hand.

For our participants, the majority of transactions were executed mentally and solutions were delivered orally without resorting to paper and pencil or any other computational tool to facilitate their problem solving. As such, the participants relied more on manipulating quantities (i.e., produce-vendors, tickets-conductors), rather than manipulating symbols to conduct their work-related activities. However, when asked to write their solutions, there was no evidence, once numbers are written down, that the participants related their obtained answers to the problem at hand in order to assess the adequacy of their answers.

For example, both vendors and conductors repeatedly obtained different solutions when solving the same problem using mental intuitive strategies at one time and written idiosyncratic computational strategies at another. That is to say, their mental intuitive solution strategies existed separately from their knowledge of written computations. They did not relate the solutions to problems in real world setting with the answers they obtained when solving the same problem using paper and pencil. In fact, when solving word problems and writing number sentences for them; the subjects believed that it was acceptable to get a different answer to the word problem when solving it mentally than when solving it algorithmically in written form.

For instance, a vendor gave two completely different answers when solving the following problem mentally than when using written symbols:

If a shirt costs 8500 L.L. and a pair of trousers 11000 L.L. then by how much did the cost of the trousers exceed that of the shirt?

When solving this problem mentally, the vendor quickly gave the answer 2500 L.L. When asked to represent his solution in written form, he accurately used the subtraction operation; however, he was not successful in completing the traditional subtraction algorithms. On a first trial, he placed the numbers as follows:

```
 11000
        -
 8 500
-------
```

When asked about the reason for placing numbers in this way, he said:" It doesn't matter, it works both ways". Then looking at the most significant digits in the two numbers, he thought it will not work so he reversed the subtrahend and minuend and this was his second trial where he wrote:

```
  8500
        -
 11000
-------
 74  00
```

He started from right-to-left, using column subtraction, ignoring a zero in the subtrahend he said:" 0 take away 0 gives 0, 0 take away 0 is 0, 5 take away 1 gives 4 and 8 take away 1 gives 7."

A conductor-participant was presented the following problems and asked to solve it both orally and using written strategies. Using oral strategy, he determined the answer to this problem in less than 10 seconds.

> Four members of a family want to travel to destination A. The ticket price to travel to destination A is Rs.5.30. One passenger initially gives Rs.15 towards the total ticket cost. How much more does the passenger need to pay?
>
> Oral strategy: First I calculated 4 * 5 to get 20 (in rupees). Then find 4 * 30 (in coins) to get 1.20. The total ticket fare is 21.20. However, the passenger gave just 15 rupees. To get to 21.20, first I add 5 to 15 to get to 20; then add another 1.20 to get to 21.20. So the deficiency is 5+1.20 = 6.20
>
> Written Strategy: The conductor wrote down the problem in two steps as follows.
>
> a. First, find 5.30 * 4.
> b. Then find 21.20 – the answer from (a).

In order to calculate the answers to part a, he wrote the following statement in his paper.

$$\begin{array}{rr} & 5.30 \\ * & \underline{\quad 4} \\ & \underline{\quad\quad} \end{array}$$

He solved the problem using a traditional approach. Although he solved the problem correctly, he took more than 30 seconds to determine the answer as he was confused about the placement of the decimal point. This conductor likened the 'written approach' to the school-type problems and he attempted to use the traditional school-taught strategies, often with little success. The conductor cited the context (bus conducting) and related parameters (monetary units) as influential factors for his success in oral computation. He ignored these same parameters while using the written approach because he likened this approach to the school-type problems and hence attempted to use traditional school-taught strategies, which he was unable to recall instantly.

Given the inconsistency between calculating using symbols and their knowledge when quantities or other referents such as money were involved, it seemed that vendors were more confident that their mental approaches were more meaningful than their written ones; consequently, they resorted more often to using informal intuitive computational strategies.

Additionally, we looked at the pragmatic property of the representation system that practitioners used when reflecting on the solution obtained and addressing the need to extend their thinking to fit the environment in which the problem emerged. For the vendors as well as for the bus conductors, mathematics is inextricably linked to their daily practice and becomes a part of a broader scheme where solving a mathematics- related problem is just a part of solving the larger problem of executing a work-related task that serve some specific function.

More often, the mathematics that both practitioners encountered in the workplace has unique characteristics at odds with conventional mathematics. To illustrate, bus conductors continuously reformulate their preset goals to adapt to unexpected incident in their daily practice, such as public strikes, frequent checking inspections, and accidents. Here's an excerpt of a situation where a bus conductor reshaped some of his work-related goals.

There was this technical problem and the bus could not be operated further. It was only the third single on that shift. I knew that I could not complete the remaining 7 trips on that shift. I could not achieve the target of 2000 rupees. At that time, my only thought was to arrange for alternate travel arrangements for my passengers.

Examining the representation systems employed by street vendors and bus conductors it was obvious that both groups of practitioners exploited their external work environment to support their problem-solving behavior and drew on available modal resources to make meaning in specific contexts. As such, practitioners created multimodal patterns of behavior that were associated with specific situations and used these patterns as prompts for cognitive processing. Such multimodality in communication including speech, gestures, body posture that was prevalent in both work settings is part of the meaning making process that both vendors and bus conductors experienced in their everyday work contexts. We argue that practitioners' immersion in their work settings and their intensive interaction with it facilitated the construction of mental representations that practitioners conceptualized and appropriated into routines to address problems that emerged in their daily practice.

Comparison 3: Street vending and bus conducting heuristics-in-action

Vergnaud (1988) maintained that all "mathematical behaviors" are tied to certain mathematical concepts and that the existence of these concepts does not necessarily mean that subjects are fully aware that they are behaving accordingly. We call these mathematical behaviors as heuristics-in-action and define heuristics-in-action as the ways the in which practitioners used the mathematical properties or relationships to resolve a problem or complete a task that emerged in their work settings. In other words, we extracted the logical steps that constituted practitioners' problem solving schemes and clustered them under heuristics-in-action. A close examination of the heuristics-in-action employed by vendors and bus conductors when attempting transactions in their work settings uncovered some school-related mathematical concepts that have been appropriated by situation and problem type (see Figures 5 & 6).

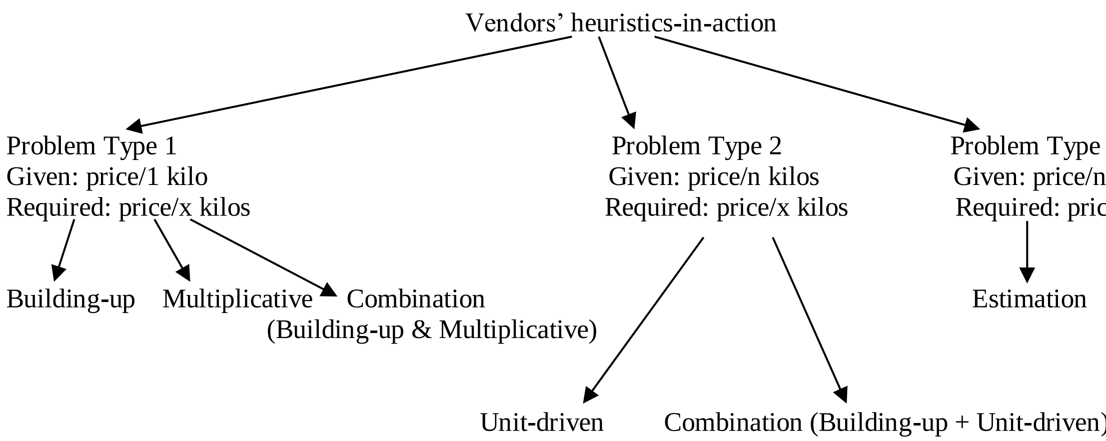

Figure 5. Classification of street vendors' heuristics-in-action by problem type.

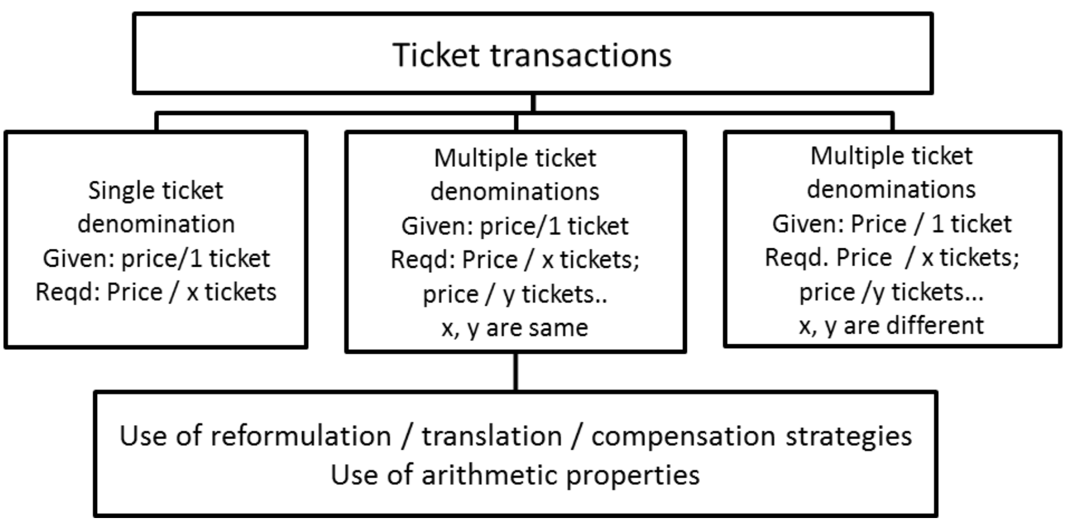

Figure 6. Classification of conductors' heuristics-in-action by problem type.

Attempting to execute a transaction in an everyday work-setting, participants were actually faced with solving a missing value proportion problem, when given the price of a certain number of units (i.e., kilos, tickets) the vendor had to calculate the price of a required number of units. Two major heuristics-in-action were employed by the participants to reach a satisfactory solution, namely Building-up and Multiplicative which in turn led to scalar and functional solutions. These heuristics are virtually based on the properties of linear functions, specifically isomorphic and functional properties (Vergnaud, 2000). To illustrate, consider the following transactions that were extracted from the two contexts:

Street Vending:

<u>*Transaction 1:*</u> This time, we approached Masri selling onions, 750L.L/ 1 kilo:

> I: "I want 3 kilos of onions, how much do I owe you?"
>
> M: "2 kilos for 750 L.L plus 750L.L which gives 1500L.L, and another 1 kilo for 750L.L then 2250L.L".

Let us analyze transaction 1 mathematically. We viewed the problem posed as one of multiplication, precisely 750x3. However, Masri did not multiply using the standard algorithm; rather, he solved the problem mentally through a *building-up* heuristic involving repeated additions which could be formalized as follows:

$$\begin{array}{rcl} 1\text{ kilo} & \longleftrightarrow & 750\text{ L.L} \\ 1+1 = 2\text{ kilos} & \longrightarrow & 750+750=1500\text{ L.L} \\ 2+1 = 3\text{ kilos} & \longrightarrow & 1500+750= 2250\text{L.L.} \end{array}$$

In this *building-up* heuristic, the variables weight and price remain independent of each other and parallel transformations are carried out on both variables thus maintaining their values proportional. When selling at a price X, the vendor was aware that an increase in the number of kilos corresponds to a proportional increase in the price (i.e., as many Xs are increased in the price as kilos are increased in the purchase). The solution thus obtained has been termed by Vergnaud (2000) as *scalar solution*. Representing the above solution formally or explicitly:

Cost(3kilos)=Cost(1kilo+1kilo+1kilo)=Cost(1kilo)+Cost(1kilo)+Cost(1kilo)=3xcost(1kilo)

If we propose a relation between weight and price, more precisely a mapping f: to every weight there corresponds a well-defined price, then the above expression can be formalized into an equality (Ø) as follows:

$$f\,(1+1+1) = f(1) + f(1) + f(1)$$

More generally, f(X+Y) =f(X) + f(Y), which Vergnaud (1988) describes as the *isomorphic property* of addition, or more specifically, the linear property of function f.

<u>*Transaction 2*</u>

The following exchange occurred between the researcher posing as a customer and Masri, a 12- year old vendor:

> I: I will take 6 kilos of lemon, how much do these cost?
>
> M (quickly): "1 kilo for 1250 L.L., if 1 kilo cost 1000 L.L then 6 kilos will cost 6000 L.L and 6 of 250 LL. Will be 1000... then 7500 L.L"

The second transaction represents a different problem solving strategy employed by the same vendor, namely multiplicative heuristic which can be also formalized as follows:

$$\begin{aligned} \text{1 kilo} &\longrightarrow \text{1250 L.L} \\ \text{6 kilos} &\longrightarrow 6(1250) = [6(1000 + 250)] \text{ L.L} \\ &\qquad = [6(1000) + 6(250)] \text{ L.L} \\ &\qquad = [6000 + (4+2)(250)] \text{ L.L} \\ &\qquad = [6000 + 4(250) + 2(250)] \text{ L.L} \\ &\qquad = [6000 + 1000 + 500] \text{ L.L} \\ &\qquad = 7500 \text{ L.L.} \end{aligned}$$

Here, Masri's solution method can be conceived in terms of a variable f(X), the price, as a function of a variable X, the number of kilos, and hence a relation can be formed through multiplying the value of X by a constant, unit price, in order to find the value of f(X). The solution obtained is called functional for it relates to two different variables, the ratio thus attained is termed "intensive" or "external" ratio (L.L/ kilo). Using the preceding argument:

Cost (6 kilos) = Cost 1 kilo *6 kilos

More generally: f(X) = f (1) * X

If we assume that the cost of 1 kilo = f (1) = constant a, then the above expression can be replaced by the following:

f(X)= a*X which is the constant function coefficient (Vergnaud, 1988).

Also, in Masri's case, (Ø) can be expressed as: f(3*1)= 3 * f(1) , or more generally f(n* x)= n*f(x) which is the isomorphic property of multiplication (Vergnaud, 1988).

***Bus Conducting*:**

Transaction 1:

A passenger approached the conductor requesting 4 tickets for destination A and 2 tickets for destination B.

> P: I want 4 tickets from to Sayani (exit point) and 2 tickets to the Sanitarium (a different exit point)
>
> C: unit ticket price to Sayani is 3.75 so 4*4 is 16… take away 4 quarters, so the price is 15; unit ticket price to sanitarium is 4.25… so 4*2 is 8 and add 50 to it to get 8.50. The total fare is 15 +8 is 23 … add another 50 to it. Give me 23.50.

The conductor broke this problem into three smaller problems:

(a) Find 3.75 * 4

(b) Find 4.25 * 2

(c) Add the answers from (a) and (b).

In this transaction, we analyze the conductor's solution as follows:

Cost of four Rs. 3.75 tickets and two Rs. 4.25 tickets = Cost of four Rs. 3.75 tickets + Cost of two Rs. 4.25 tickets = cost of one Rs. 3.75 ticket * 4 + cost of one Rs. 4.25 ticket* 2.

In other words, the above equality corresponds to: (3.75*4 + 4.25*2) = (3.75*4) + (4.25 *2) = 3.25 * 4 + 4.25 * 2

In the bus conductor's case, (Ø) can be expressed as g (3.75*4 + 4.25*2) =g (3.75*4) + g (4.25*2) = 3.25 * g (4) + 4.25 * g (2) or more generally as g (aX + bY) = a g (X) + b g (Y). Here, we can conceive the conductor's solution in terms of g (aX + bY), the price, as a function of the variables X and Y, the number of tickets two different ticket denominations with unit prices a and b respectively. Thus we form a relation by multiplying the values of X and Y by constants a and b.

<u>Transaction 2:</u>

The following exchange occurred between a passenger and a conductor on bus number 7B.

> P: Give me 3 tickets to Anna Nagar bus depot (exit point).
>
> > C (on-the-fly transaction – duration less than 10 seconds): First conductor determines the ticket fare for one passenger as 4.50 (using the entry and the exit points and by recalling the ticket fare schedule). 1ticket costs Rs.4.50. Then 3 tickets will cost 3 * 4 = 12 and half of 3 (which is 1.50). The passenger pays Rs. 13.50.

This transaction can be also formalized as follows:

1 ticket → Rs. 4.50
3 tickets → Rs. 3 (4.50)
= Rs. [3(4 + 0.50)]
= Rs. [3 (4) + ½ (3)]
= Rs. [12 + 1.50]
= Rs. 13.50

Using the preceding argument:

Cost (3 tickets) = Cost 1 ticket *3 tickets

More generally: g (X) = g (1) * X

If we assume that the cost of 1 ticket = g (1) = constant (4.25), then the above expression can be replaced by the following: g (aX) =a g(X) which is the constant function coefficient (Vergnaud, 1988).

To summarize, what we had here is in fact four simple arithmetic problems that deal with important mathematical concepts. Using Vergnaud's model as a theoretical framework for analyzing practitioners' problem solving at work, one thing was clear namely, the fact that the participants did use common heuristics-in-action (i.e., in their understanding of simple proportional relationships, a model which Vergnaud terms "the isomorphism of measures model of situations"). When using Building-up heuristic, practitioners maintained the proportionality between the values by carrying parallel transformation on the variables without dividing or multiplying values in one variable by values in the other variable. It is worth mentioning here that the rule of three, the algorithm learned in school to solve simple proportional problems, differs from the isomorphism schema because it involves the multiplication of values across variables instead of parallel transformation on the variables. Hence, practitioners employed concepts that challenged the rule-of-three algorithm taught in school today clearly preferring the use of multiplicative heuristic because of its strong ties to problem situations, which led to functional solutions.

The street vendors and bus conductors have developed mathematical concepts as a result of immersion experiences in everyday situations. The work setting represented vendors and bus conductors' natural habitats and thus introduced familiar problems. As a result, practitioners systematically and smoothly built up their solutions using intuitive computational strategies and without losing track of the strategy, even if many numbers are involved. In other words, the practitioners kept the meaning of the problem in mind during problem solving. This understanding that the practitioners acquired in the work situation elicited a coherent problem-solving behavior that was attained through the following steps: (a) translating the problem from its real life context into an appropriate mathematical calculation problem, (b) performing the mathematical calculations, and (c) translating the result of this calculation back into the context of the problem to see whether it made sense.

DISCUSSION

In this paper, we have argued that a good deal of vendors and bus conductors' work is not only physical but also conceptual, social, and interactive. The mathematics developed by the street vendors and bus conductors had some unique characteristics that were different from the mathematics that we are accustomed to seeing in textbooks and that showed how the workplace had stamped the mathematical ideas that were developed there. Workplace setting provided the practitioners with challenges and opportunities to develop meaningful mathematical understanding, which in turn led to effective problem solving on simple arithmetic problems. As a result, practitioners acquired a repertoire of meaningful heuristics and self-invented computational strategies. The main characteristic of these informal, self-invented computational strategies is meaningfulness and appropriateness to the work situations. Because the principal motive of practitioners is to make sales and do their business, their rules for solving problems at work may not be, principally, mathematical ones. Nevertheless and despite their relatively minimal level of schooling, practitioners were successful at distancing themselves from the characteristics of the sales situations in which they work and were creative in using heuristics-in-action to analyze their sales in terms of more abstract mathematical notions. As a result the mathematical ideas collectively used in the workplace are actualized through the practitioners' actions and appropriated by the situations that emerged in everyday routines. These ideas were strictly utilitarian and developed for the purpose of appropriating selling efforts to maximize profit and minimize loss. In other words, the mathematics that practitioners collectively employ emanates from their labor that is concealed within the object of production; it is always there, useful, useable and meaningful.

REFERENCES

Amsterdam, A. G., & Bruner, J. S. (2000). *Minding the law.* Cambridge, MA: Harvard University Press.

Bessot, A., & Ridgeway, J. (Eds.). (2000). *Education for mathematics in the workplace*. Dordrecht, Netherlands: Kluwer Academic Publishers.

Bruner, J. (1986). *Actual minds, possible worlds*. Cambridge, MA: Harvard University Press.

Carraher, T. N, Carraher, D. W., & Schliemann, D. S. (1985). Mathematics in streets and schools. *British Journal of Developmental Psychology, 3,* 21-29.

Ceci, S. J., & Roazzi, A. (1994). The effects of context on cognition: Postcards from Brazil. In R. J. Sternberg & R. K. Wagner (Eds.), *Mind in context: Interactionist perspectives on human intelligence* (pp.74-101). New York: Cambridge University Press.

Coulter, C., A., & Smith, M. L. (2009). The construction zone: Literary elements in narrative research. *Educational Researcher, 38*(8), 577-590.

D'Ambrosio, U. (1992). Ethnomathematics: A research program on the history and philosophy of mathematics with pedagogical implications. *Notices of the American Mathematical Society, 39*(10), 1183-1185.

Dawe, L. (1988). *Mathematics teaching and learning in village schools of the South Pacific*. Paper presented at the Sixth International Congress on Mathematical Education, Budapest.

Gay, J., & Cole, M. (1967). *The new mathematics and an old culture: A study of learning among the Kpelle of Liberia.* New York: Holt, Rinehart & Winston.

Gerdes, P. (1998), *Women, art and geometry in Southern Africa*. Lawrenceville, NJ: Africa World Press.

Ginsburg, H. P. (1988). *Children's arithmetic: The learning process*. New York: Van Nostrand.

Hochestin, E. (2008). Endorsing non-occurrent beliefs: Why standard belief functioning is problematic for the extended mind. In B. Hardy-Vallee & N. Payette (Eds.), *Beyond the brain: Embodied, Situated and Distributed Cognition* (pp. 205-214). Newcastle, UK: Cambridge Scholars Publishing.

Hoyles, C., & Noss, R. (2010). *Improving mathematics at work.* London, UK: Routledge

Hoyles, C., Wolf, A., Molyneux, Hodgson S. & Kent, P. (2002). *Mathematical skills in the workplace: Final report to the Science, Technology and Mathematics Council.* London: Institute of Education, University of London/STM Council.

Jurdak, M., & Shahin, I. C. (2002). Problem solving activity in the workplace and the school: The case of constructing solids**.** *Educational Studies in Mathematics*, 47, 297-315.

Jurdak, M., & Shahin, I. C. (1999). An ethnographic study of the computational strategies of a group of young street vendors in Beirut. *Educational Studies in Mathematics Education, 40*, 155-172.

Kearins, J. M. (1981). Visual spatial memory in Australian indigenous children of the desert regions. *Cognitive Psychology*, *13*, 434–460.

Kirsh, D., & Maglio, P. (1995). On distinguishing epistemic from pragmatic actions. *CognitiveScience*. 513-549.

Lakoff, G., & Nùňez, R. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being.* New York: Basic Books.

Lave, J. (1988). *Cognition in practice*. Cambridge, U.K.: Cambridge University Press.

Lindlof, T. R., & Taylor, B. (2002). *Qualitative communication research methods.* Thousand Oaks, CA: SAGE Publications, Incorporated.

Magajna, Z., & Monaghan, J. (2003). Advanced mathematical thinking in a technological workplace. *Educational Studies in Mathematics*, *52*(2), 101-122

Masingila, J. O. (1994). Mathematics practice in carpet laying. *Anthropology and Education Quarterly, 25*(4), 430-462.

Masten, A., & Coatsworth, D. (1995). Competence, resilience, and psychopathology. In D. Cicchetti & D. Cohen (Eds.), *Developmental Psychopathology -Volume 2: Risk, Disorder, and Adaptation* (pp. 715-752). New York: John Wiley & Sons, Inc.

Mercer, N**.** (2000) *Words and minds: How we use language to think together.* London, UK: Routledge.

Millroy, W. L. (1992). An ethnographic study of the mathematical ideas of a group of carpenters. *Journal for Research on Math Education Monographs, 5* (0883-9530).

Moore, N. (2006). The contexts of context: Broadening perspectives in the (re)use of qualitative data**.** *Methodological Innovations Online*, *1*(2), 21-32

Naresh, N. (2008). *Workplace Mathematics of the Bus Conductors in Chennai, India.* Unpublished doctoral dissertation. Illinois State University.

Noss, R., Hoyles, C., & Pozzi, S. (2000). Working knowledge: Mathematics in use. In A. Bessot & J. Ridgway (Eds.), *Education for mathematics in the workplace* (pp. 17-35). Dordrecht, Netherlands: Kluwer Academic Publishers

Nunes, T., Schliemann, A. D., & Carraher, D. W. (1993). *Street mathematics and school mathematics.* Cambridge, UK: Cambridge University Press.

Reichelt, A., & Rossmanith, N. (2008). Relating embodied and situated approaches to cognition. In B. Hardy-Vallee & N. Payette (Eds.), *Beyond the brain: Embodied, situated and distributed cognition* (pp. 57-67). Newcastle, UK: Cambridge Scholars Publishing.

Sanin, C., & Szczerbicki, E. (2009). Application of a multi domain knowledge structure: The decisional DNA. In *Intelligent systems for knowledge management* (pp. 65-86). Dusseldorf, Germany: Springer Verlag.

Saxe, G. B. (1988). Candy selling and mathematics learning. *Educational Researcher, 17*(6), 14-21.

Saxe, G. B. (1991). *Culture and cognitive development*: *Studies in mathematical understanding*. Hillsdale, NJ: Lawrence Erlbaum Association.

Standler, R. (2000). *Mathematics for engineers.* Retrieved from http://www.rbs0.com/math.htm

Vergnaud, G. (1988). Multiplicative structures. In M. Hiebert & J. Behr (Eds.), *Number concepts and operations in the middle grades* (pp.46-65). Reston, VA: National Council for Teachers of Mathematics.

Vergnaud, G. (2000). Introduction. In A. Bessot, & J. Ridgeway (Eds.), *Education for mathematics in the workplace*. (pp. xvii-xxiv). Dordrecht: The Netherlands. Kluwer Academic Publishers.

Wang, Q., Ceci W. M., Williams K., & Kopko, A. (2004). Culturally situated cognitive competence: A functional framework. In R. Sternberg (Ed.), *Culture and competence* (pp. 225-249). Washington, DC: American Psychological Association.

Wenger, E. (2000). Communities of practice: The organizational frontier. *Harvard Business Review*, *78*(1), 139-145.

Zevenbergen, R. (2000). Research methods for mathematics at work. In A. Bessot & J. Ridgway (Eds.), *Education for mathematics in the workplace* (pp. 181-188). Dordrecht, Netherlands: Kluwer Academic Publishers.