Análisis de pruebas de presión en yacimientos fracturados fractales

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Resumen

Este artículo presenta un método para analizar pruebas de variación de presión con rasgos diferentes de los más comunes que denotan geometrías de flujo lineal, radial o esférico, y que son indicadores de flujo de tipo fractal en el yacimiento.

El método utiliza una solución a la respuesta de presión con flujo de tipo fractal en el dominio de Laplace e incluye los efectos de daño y almacenamiento por medio de un procedimiento usado para la descripción del flujo en geometrías compuestas. Este enfoque es más sencillo que los presentados en artículos anteriores a éste.

La determinación de las propiedades fractales en aplicaciones a casos de campo da por resultado valores plausibles; para la comparación con valores reales es necesario conocer la distribución de las fracturas en el yacimiento.

Palabras clave: Pruebas de variación de presión, fracturas, fractal.

Analysis of well-tests in fractal-fractured reservoirs

Abstract

This paper presents a method to analyze well tests with traits that deviate from the usual log derivative pressure plots -manifest in linear, radial or spherical geometries- and point to the presence of fractal flow geometries in a reservoir.

The method uses a fractal geometry pressure response solution in the Laplace domain and includes skin and wellbore fill up effects in a scheme used previously for flow in composite geometries. The approach is simpler than previous schemes.

Determination of fractal properties from applications to field cases result in plausible values; but comparison with actual values requires maps of the fractures distribution.

Keywords: Well tests, fractures, fractal.

Introduction

The main characteristic of transient flow in fractal media is that the pressure time-log derivatives (see Tiab and Kumar, 1980) do not present the values ½, 0 or -½ typical of corresponding linear, cylindrical or spherical flow geometries. The intermediate values in pressure transients correspond to fractional exponents in the diffusion equation that represent the "fractality" of the media.

Porous media show a disordered structure in pore and rock spatial distributions; then, why non-classical slopes show in only a small fraction of well tests? Studies of unconsolidated soils conclude that only 20% of the soil samples present such fractality, (Gimenez, Perfect e al. 1997), (an equivalent study in oil or water reservoir rocks has not been found by the authors). On the other hand, an abundance of fractal regions results in a normal or Gaussian transient flow behavior because of the *central limit theorem*: the sum of several probabilistic distributions of any type amounts to a normal distribution (see Vlahos, Isliker et al. 2008). That is, no matter how complex the porous medium the transient fluid flow is equivalent to a normal pressure evolution –with its corresponding regular value of flow-geometry slope in the log-time derivative plot; paradoxically, a high degree of heterogeneity results in an overall homogeneous, regular, flow.

Analysis of well tests in reservoirs with fractal porous media was initiated by the work of O´Shaugnessy and Procaccia (1985), they present a diffusion equation with regular time derivative and a space derivative term that stems from scaling arguments.

$$\partial^{\beta} p/\partial^{\beta} t = \frac{1}{r^{\alpha}}\frac{\partial p}{\partial r}\left(\eta_0 \frac{1}{r^{\alpha}}\frac{\partial p}{\partial r}\right) \tag{1}$$

The regular normal or Gaussian solutions apply when β=1, α=d-1. *d*=dimension. The aforementioned authors assumed β=1, α=ds-1, where *ds* represents the fractal medium characterized by $d_s=2d_f/d_w$; *df* is the static exponent; *dw* the dynamic exponent.

The practical aspects of skin and wellbore storage through use of auxiliary equations dependent on radial and time derivatives,

$$\Delta p_{skin} \sim s\left(\frac{\partial p}{\partial r}\right)_{r_w} \tag{2}$$

$$q_{well-head} = q_{well-bottom} + C\frac{\partial p}{\partial t} \tag{3}$$

(Agarwal, Hussainy et al. 1970), were incorporated by Chang and Yortsos (1990) to obtain, in the Laplace domain,

$$\tilde{p}_{wD}(\tau) = \frac{K_{\nu}(\zeta)+s\zeta K_{\nu-1}(\zeta)}{\tau\{C_D\tau\zeta[K_{\nu}(\zeta)+s\zeta K_{\nu-1}(\zeta)]+\zeta K_{\nu-1}(\zeta)\}} \tag{4}$$

Where ζ and √τ = Laplace parameter. Zhao and Zhang (2011), assumed skin and storage as implicit in the integration constants and obtained a result similar to eqn. (4).

The Chang and Yortsos (1990) approach has been criticized as inaccurate (Sahimi 1995, Camacho, Fuentes et al. 2008) due to its failure to describe interference tests (with high *r* and *t* values) originated by the exclusion of the time-fractal derivative. Besides, it is highly improbable for any fractal medium to present fractal-space characteristics (number of nodes –also called "mass"- dependence on a fractional negative power *df* of *r*) but no fractal-time, dynamic behavior. The later is the outcome of a random walk in the medium with the involved geometry described by its static exponent; (see Vlahos, Isliker et al. 2008).

In several media the ratio of dynamic to static exponents is close to 3/2 (Alexander and Orbach, 1982) although deviations from that ratio have been reported. In this work we assume that ratio as valid.

An improvement in the fractal diffusion formulation was presented by Metzler, Glockle et al. (1994), from now on "MGN". Their diffusion equation includes a fractal-time derivative, with exponent β = 1/dw, but preserves the scaling approach of O´Shaugnessy and Procaccia (1985) in the space term. For a unit impulse at *r=0* the MGN solution is,

$$\tilde{p}_{wD}(r,\tau) = \left(\frac{r}{\tau^{\beta}}\right)^{\nu} K_{\nu}\left(r\tau^{\beta}\right) \qquad (5)$$

Where $\nu=1-d_s/2$. Notice that when β=1 and ds=dimension, equation (5) is the usual instantaneous line source solution.

The MGN improvement results in the ability to describe interference tests (de Swaan, Camacho-V et al. 2013). The approach is still an approximation to the fully time-space fractal formulation that originates in probabilistic considerations, (see Montroll and Weiss (1965), Compte (1996) and Vlahos, Isliker et al. (2008)). Notice that probability of the position in space of one particle can be transformed to density when multiplied by the number of particles in a system; density is transformed directly into pressure for low compressibility fluids.

The MGN eqn. (5) was extended to include skin and storage by Park, Choe et al. (2001) by using the auxiliary equations (2) and (3), it resulted in the equation,

$$\tilde{p}_{wD}(z) = \frac{K_{\nu}(z^{\gamma})+s[z^{\gamma}K_{\nu+1}(z^{\gamma})-2\nu K_{\nu}(z^{\gamma})]}{z[z^{\gamma}K_{\nu+1}(z^{\gamma})-2\nu K_{\nu}(z^{\gamma})]+z^{2\gamma+1}C_D\{K_{\nu}(z^{\gamma})+s[z^{\gamma}K_{\nu+1}(z^{\gamma})-2\nu K_{\nu}(z^{\gamma})]\}} \qquad (6)$$

where γ *=1/dw.*

In eqns. (4) and (6) modified Bessel functions *K* of order different from ν arise as derivatives *dp/dr* to account for flow.

In the present paper the formulation with fractional time, the MGN approximate solution is assumed as representative of the pressure behavior in the reservoir but includes skin and storage in a practical as well as convenient way previously used to describe flow patterns in composite reservoirs, (see de Swaan 1998).

Theory

The formulation is made in the Laplace domain with parameter τ. The symmetric transform is made numerically, (Stehfest 1970).

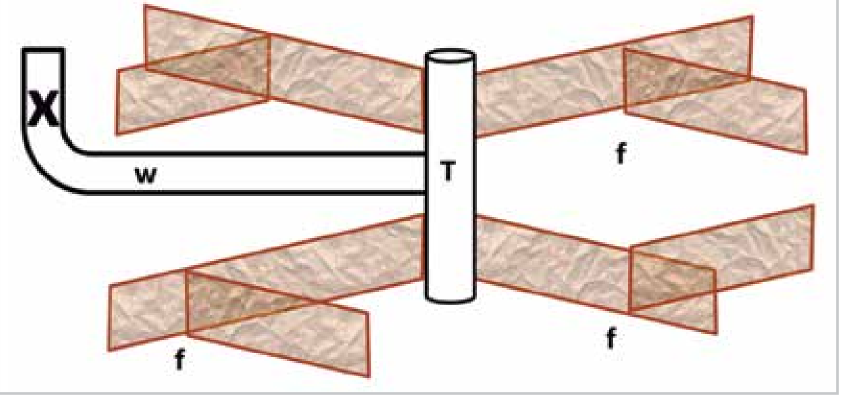

Figure 1. Horizontal well, ***w***; valve, **X**; transition zone, **T**; and fractures, **f**.

The compound flow trajectory, from reservoir to wellhead, shown schematically in **Figure 1**, is assumed as composed of the following connected zones:

1) The wellbore that includes storage and skin. Although the detailed pressure in the tubing as a linear medium can be included, due to its high transmissivity it is assumed as a tank with instantaneous pressure changes along its length. Connected fractures to the wellbore are not present along the well length. The pressure at the well head is the pressure at the extreme of the well minus the pressure drop due to skin; at the same time, the velocity at the head is the unitary rate of flow at the bottom plus the storage effects in the tubing volume which depend on the rate of change *dp/dt* in the well, (Agarwal, Hussainy et al, 1970).

$$\begin{pmatrix}\tilde{p}_w\\ \tilde{v}_w\end{pmatrix} = \begin{pmatrix}1 & -s\\ C\tau & 1\end{pmatrix}\begin{pmatrix}\tilde{p}_T\\ \tilde{v}_T\end{pmatrix} \quad (7)$$

Between neighboring zones multipliers of velocity of the form A_1/A_2 are necessary to account for velocity changes due to different surface areas transverse to flow. Each velocity change is proportional to the ratio of areas.

2) Flow into an intermediate –non fractal- zone with some regular geometry. This point stems from the existence of a limited-connection zone between well and fractal reservoir region. Similarly, it has been experimentally observed that induced fractures made in outcrops, far from connecting fully with the fractures, show a transition zone between the well and the fracture proper, (Suarez-Rivera, Behrmann et al. 2013). In the present work, that intermediate zone between well and fractured reservoir is represented as a vertical cylinder; radius rT. It is represented by a matrix *T* that in general may have the properties of a regular -non fractal- geometry zone (de Swaan, 1998). In the cylinder it only affects the change of velocity proportional to the ratio of areas of transition zone and well.

$$\begin{pmatrix}\tilde{p}_T\\ \tilde{v}_T\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & A_T/A_w\end{pmatrix}\begin{pmatrix}\tilde{v}_f\,\tilde{p}_f/\tau\\ \tilde{v}_f\end{pmatrix} \quad (8)$$

3) The right term vector includes the pressure and velocity vector in the fractal reservoir, which depends on the fractal geometry; $\tilde{p}_f$ is described by the MGN solution, equation (5) and the transformed velocity $\tilde{v}_f$ is indeterminate as yet. That solution is for a unit delta impulse at the origin. It is then integrated in time in the Laplace domain (factor *1/τ*) and convolved with the flow velocity at the fractures face with transform $\tilde{v}_f\ \tilde{p}_f$.

The final formulation involves the sequential product of matrices representing each stage in the flow path, (see de Swaan 1998). Both *p* and also *v* (or *q*) are unknowns at each boundary between stages in the flow path. In contrast with the results of previous formulations presented in the introduction, it is not necessary to formulate *v* in terms of *dp/dr* but it is handled as an unknown.

$$\begin{pmatrix}\tilde{p}_w\\ \tilde{v}_w\end{pmatrix} = \begin{pmatrix}1 & -s\\ C\tau & 0\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & A_w/A_T\end{pmatrix}\begin{pmatrix}\tilde{v}_f\,\tilde{p}_f/\tau\\ \tilde{v}_f\end{pmatrix} \quad (9)$$

$\tilde{v}_f$ is a function of $\tilde{v}_w$ the velocity at the well head. $\tilde{p}_f$ is the MGN solution and $\tilde{p}_w$ is obtained from both variables in the system of two equations represented by the matrix product and vectors.

Results

Comparison of type curves obtained from the resulting equations and after applying the Stehfest (1970) transform result in curves shown in **Figure 2**. In two dimensions and for cases: a) normal *df=2, dw=2*; and b) fractal, *df=1.7, dw=2.55*. (Compare with the ones presented by Zhao and Zhang (2011) in their **Figure 1** where *dw=2;* β=1 for all cases.) The graphs are shown just for comparison purposes. (The type-curves method is obsolete given that any personal computer can not only generate functions but also automatically match them to observations in minimal times).

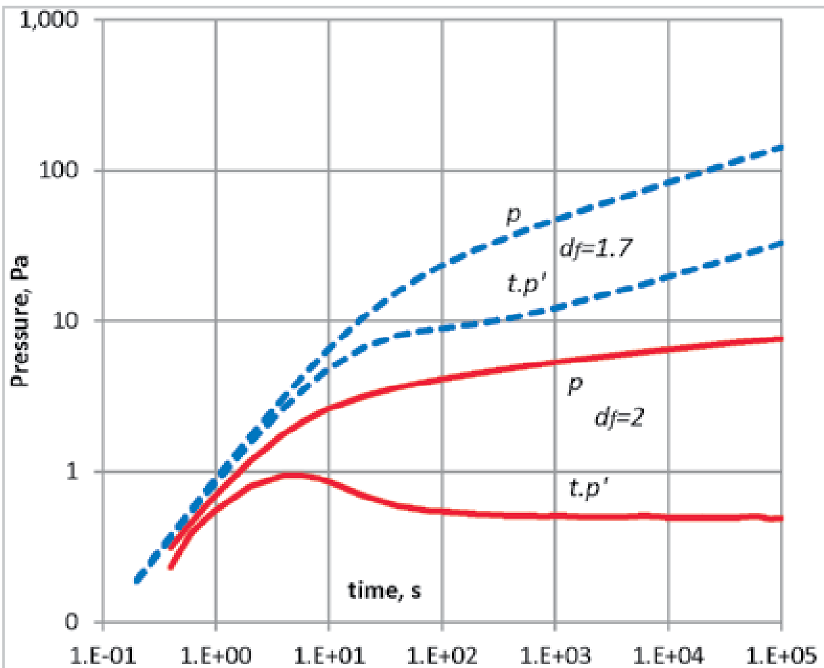

Figure 1. Pressure response and derivative, normal 2D and fractal df=1.7; C=1000, s=2.

Application to actual field cases.- The theory was applied to well tests from horizontal wells with closure at the producing depth as depicted in **Figure 1**. The reservoir is a fractured shallow limestone. Production is present only when the wells connect with fractured zones and there is no apparent production from the matrix-rock to the wellbore.

The horizontal wells intersect producing fractured zones at an average of 106 m, that distance was input as the horizontal well length to a single fractured zone.

The curves were fitted using the Minpack (1999) software.

In **Figure 2**, a sudden change in the m=1 zone is probably a measurement error.

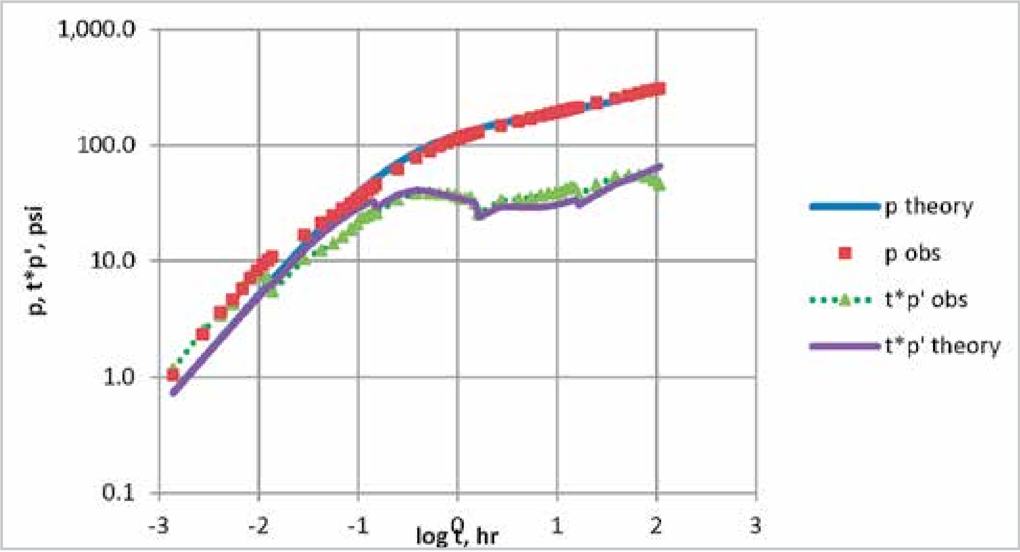

Figure 2. Well A09, observed and matched graphs of log pressure and derivative vs log time.

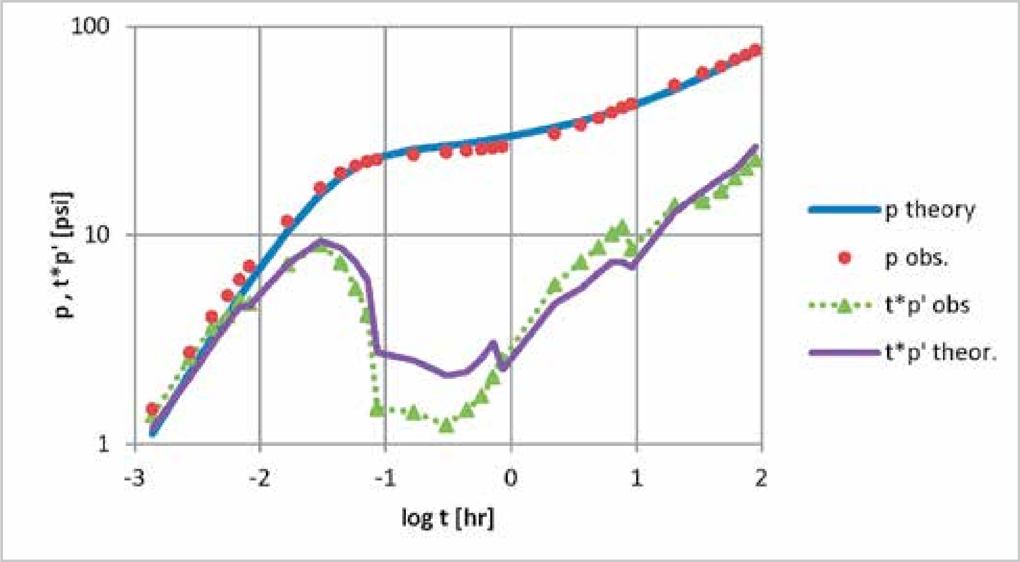

Figure 3. Well A23, observed and matched graphs of log pressure and derivative vs log time.

Table 1 contains the data used in the matches. There are no maps of actual fracture distributions in the field and no comparison is possible between matched and field fractal exponents.

Table 1

well	ϕ	***μ Pa.s***	***q m3/d***	***dw***	***df***	ν	***k***	***s***	***C***	r_T
A09	0.09	2.8	53	2.18	1.45	0.428571	1.44E-10	-1.5E+5	5E-4	2.4E-6
A23	0.07	1.8	155	2.09	1.393	0.47368	6.78E-18	-2700	9.4E-5	1.4E-6

The compressibility value used is 6.0E-9 m3/Pa. Net pay is 85 m in both cases. The transition zone is fitted with very low values of radius, rT, in both cases; that makes the pressure response in the fractured reservoir to be close to a line source –null radius- in contact with the well.

Conclusions

A method is presented to obtain fractal reservoir properties from well-tests. The method is simpler to apply than former models presented in the literature on that subject.

Application of the theory to field cases shows its feasibility; but comparison of the fractal-properties values obtained through matches with actual field values requires maps of the distribution of fractures in the tested formations.

Nomenclature

A	surface area m^2
C	storage coefficient m^3/Pa
d	dimension
ds	spectral exponent
h	thickness, m
k_v	modified Bessel function of order v
m	slope in log-log plot of pressure log-t derivative
p	pressure, Pa (psi in figs. 2 and 3)
pwD	dimensionless pressure at the well radius
q	flow rate, m^3/sec , (m^3/day)
r	radial distance, m
s	skin, Pa/m^3/s
t	time, sec (hr in Figs. 2 and 3)
x	coordinate, m
α	fractal dimension exponent
β	dynamic exponent (=)
γ	*1/dw*
ηo	average hydraulic diffusivity, $(m^2/sec)^{\alpha/\beta}$
μ	viscosity Pa.sec
τ	time-Laplace transform parameter

Subscripts and superscripts

f	fractal static dimension
w	Brownian or random-walk
s	spectral dimension
$\tilde{u}$	time-Laplace transform of *u*
T	transition zone

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