2D preconditioned Kirchhoff least-squares depth migration: An approach to true reflectivity

Ing. Alejandro Cabrales-Vargas
alejandro.cabrales@pemex.com
Ing. Alfredo Vázquez Cantú
alfredo.vazquez@pemex.com
Pemex Exploración y Producción

Información del artículo: Recibido: enero de 2013-aceptado: octubre de 2013

Resumen

La migración de Kirchhoff es todavía el algoritmo más popular debido a que fácilmente maneja geometrías de adquisición irregulares, que ocurren debido a falta de permisos y obstáculos en superficie. Sin embargo, si no se interpola apropiadamente, dichas geometrías irregulares afectan adversamente la preservación de amplitud. La migración de cuadrados mínimos pre-acondicionada (PLSM, por sus siglas en inglés) ofrece una alternativa a la interpolación 5D, y proporciona una aproximación de la reflectividad de la Tierra, que se compara con el inverso del modelado sísmico. La migración PLSM puede considerarse también como un método de interpolación basado en el modelo de velocidades del subsuelo. Su impacto en la sección apilada es mínimo; la ventaja principal consiste en la obtención de registros migrados antes de apilar, para su uso en análisis de AVO y AVAz. Otros usos potenciales de la migración PLSM incluyen la regularización de datos antes de la migración RTM, y la representación sísmica por difracciones. La principal desventaja del método es el incremento del tiempo de cómputo en un factor de 6-12.

Palabras clave: Migración Kirchhoff, migración de cuadrados mínimos, migración antes de apilar, preservación de amplitud, adquisición, regularización.

Abstract

Kirchhoff migration is still the most popular imaging algorithm because it easily handles irregular acquisition geometries, which occur due permitting limitations and surface obstacles. However, if not properly interpolated, irregular geometries also hinder proper amplitude preservation. Preconditioned least-squares migration (PLSM) offers an alternative to 5D interpolation, and provides an approximation to Earth's reflectivity that resembles the inverse of seismic modeling. PLSM can also be viewed as an interpolation method based on the subsurface velocity-depth model. The impact of PSLM on final stacked results is minimal, with the main advantage being clean prestack gathers for subsequent AVO and AVAz analysis. Other potential uses of PLSM include regularization of data prior to RTM, as well as a component of diffraction imaging. The main disadvantage of the method is the increase in computational time by a factor of 6-12.

Keywords: Kirchhoff migration, least-squares migration, prestack migration, amplitude preservation, acquisition, regularization.

Introduction

In spite of the advantages of the wave-equation methods, e.g. reverse-time migration, we still trust the Kirchhoff migration capabilities to address irregular acquisition and to recover meaningful amplitudes, the latter of paramount importance for AVO, AVAz, and elastic inversion. However, Kirchhoff migration still suffers from aliasing artifacts associated with sparse and irregularly sampled surface acquisition. Seismic migration is often regarded as the process which reverses wave-propagation during seismic acquisition, and as stated by Claerbout (1992), constitutes the adjoint operator of seismic modeling. In presence of irregular acquisitions, the assumption that migration is the inverse process of seismic modeling no longer holds. To address this shortcoming, one can minimize the difference between the acquired data and modeled data via leastsquares migration.

For many years, least-squares migration methods have been proposed and applied successfully both to synthetic and real datasets (e.g. Nemeth et al., 1999; Duquet et al., 2000; Aoki and Schuster, 2009). However, to my knowledge, least-squares migration is not currently offered by processing service companies. One explanation is the higher cost compared to conventional migration, with each iteration of least-squares migration costing about two conventional migrations. For this reason it is important to reduce the number of iterations. For example, Guo et al. (2012) found that three iterations were sufficient when applying 3D PLSM time migration to land surveys from Kansas and Oklahoma. In this paper I extend the PLSM method to depth and anisotropy.

Least-squares migration review

We represent seismic modeling as a linear system in terms of matrix theory (bold letters represent matrices or vectors):

$$d=Gm \qquad \text{...(1)}$$

Here d is the vector of modeled data, m is the vector of reflectivity, and matrix G represents the modeling operator. Direct inversion of equation (1) is unpractical because G is not square and extremely large. To approximate such an inverse I utilize least–squares migration to minimize the objective function.

$$F=||Gm-d||^2 + ||\lambda\, R_h(m)||^2 \qquad \text{...(2)}$$

Where R_h is a roughness function (e.g., a first derivative) in the offset domain (h), and λ is a penalty function that controls the amount of roughness, in our case constituted by non-flat events associated to aliasing. Minimizing the first term at the right of equation (2) yields

$$[G^T G]m \approx G^T d \qquad \text{...(3)}$$

Where G^T is the adjoint operator of modeling (conjugate transpose in matrix notation). Now G^TG is square. I label equation (3) as least-squares migration *sensu stricto*, to differentiate it from the more general case including the penalty function, regarded as constrained least-squares migration (Cabrales-Vargas, 2011). The right hand side of equation (3) represents conventional migration, which approximates the inverse of modeling when G^TG is diagonally dominant. According to Nemeth (1996), this occurs when acquisition is dense and regular, and the velocity model does not overly focus or defocus the data. Otherwise, the conventionally migrated output, $m'=G^Td$, will be corrupted by a non–diagonal matrix, which acts as a blurring operator (Hu et al., 2001; Aoki and Schuster, 2009).

$$[G^T G]m \approx m' \qquad \text{...(4)}$$

Such that the result departs from the desired reflectivity. In this case least-squares migration achieves a much better approximations. The second term at the right of equation (2) filters-out non-flat events in common-image gathers. Including such a term, the minimization becomes.

$$[G^T G + \lambda\, R_h]m \approx G^T d \qquad \text{...(5)}$$

Directly solving equation (5) requires huge amounts of memory because the vectors and matrices are big, even for 2D datasets. Hence, I employ numerical solutions to use the operators G and G^T, and avoid explicitly creating the matrices.

Preconditioned least-squares migration

Equation (5) can be implemented by preconditioning the model instead of using a constraint function, (Guo, 2012). Substituting $z = R_h m$ in equation (5) we obtain

$$[(R_h^{-1})^T G^T G R_h^{-1} + \lambda \mathrm{I}]z \approx (R_h^{-1})^T G^T d \qquad \text{... (6)}$$

The inverse of the roughness function, R_h, can be approximated by a smoothness function (Wang et al., 2004), $P_h \approx R_h^{-1}$. Making these substitutions in equation (6), and dropping λ according to Wang (2005), I obtain

$$[P_h^T G^T G P_h] z \approx P_h^T G^T d \qquad ...(7)$$

Finally, I define $G = GP_h$, and substituting into equation (7) we obtain

$$[\underline{G}^T \underline{G}] z \approx \underline{G}^T d \qquad ...(8)$$

Finally, I recover the estimated reflectivity with $m \approx P_h T_z$. Equation (8) is solved with the conjugate gradient method (Hestenes, 1973; Nemeth, 1996). In this implementation to minimize aliasing artifacts.the model preconditioned is using a three-point mean filter in every common–image gather.

Methodology

The PLSM is tested on the anisotropic Marmousi dataset. Kirchhoff migration and demigration codes utilize travel time tables previously computed with the fast marching method (Sethian and Popovici, 1999), incorporating anisotropy according to Lou (2006).

Results

The results of conventional migration versus PLSM for ten iterations in original and decimated versions of the Marmousi dataset are compared. Decimation consisted in randomly killing about 75% of the original input traces. The results are shown in representative common–image gathers, common-offset gathers, and stacked sections. The anisotropic version of Marmousi, (Alkhalifah, 1997) consists of 240 shots, with 96 traces per shot, with total recording time of 2.9 s. Shots and receivers are spaced 25 m, and time sampling is 0.004 s. Velocity and anisotropy (η) models are shown in **Figure 1**. For depth migration, Thomsen parameters are defined as ε=η and δ=0 (Alkhalifah, personal communication).

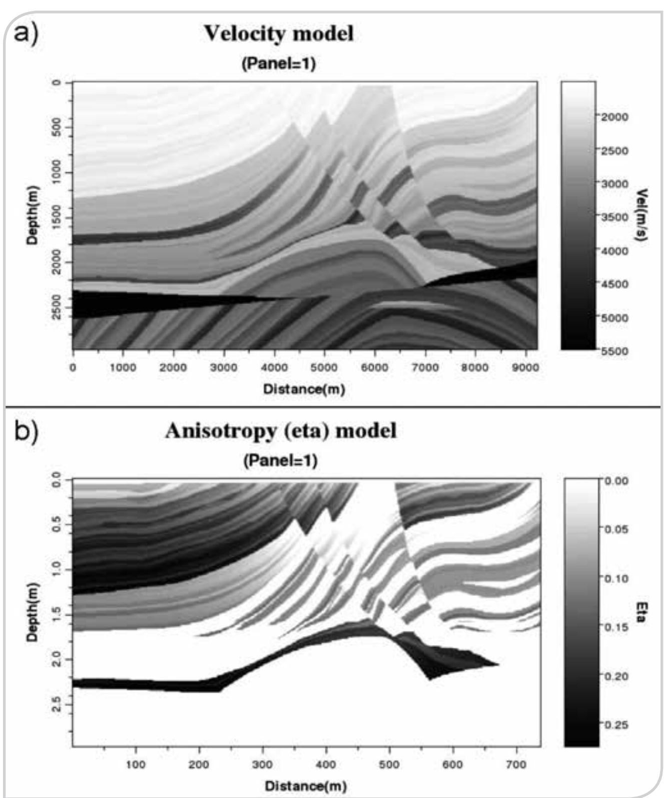

Figure 1. Marmousi model: (a) Velocity model; (b) Anisotropy η model.

Figures 2 show zooms of the original and decimated datasets. In order to avoid modeling zero traces, they are automatically excluded from the calculations. These and the following plots where equivalently clipped.

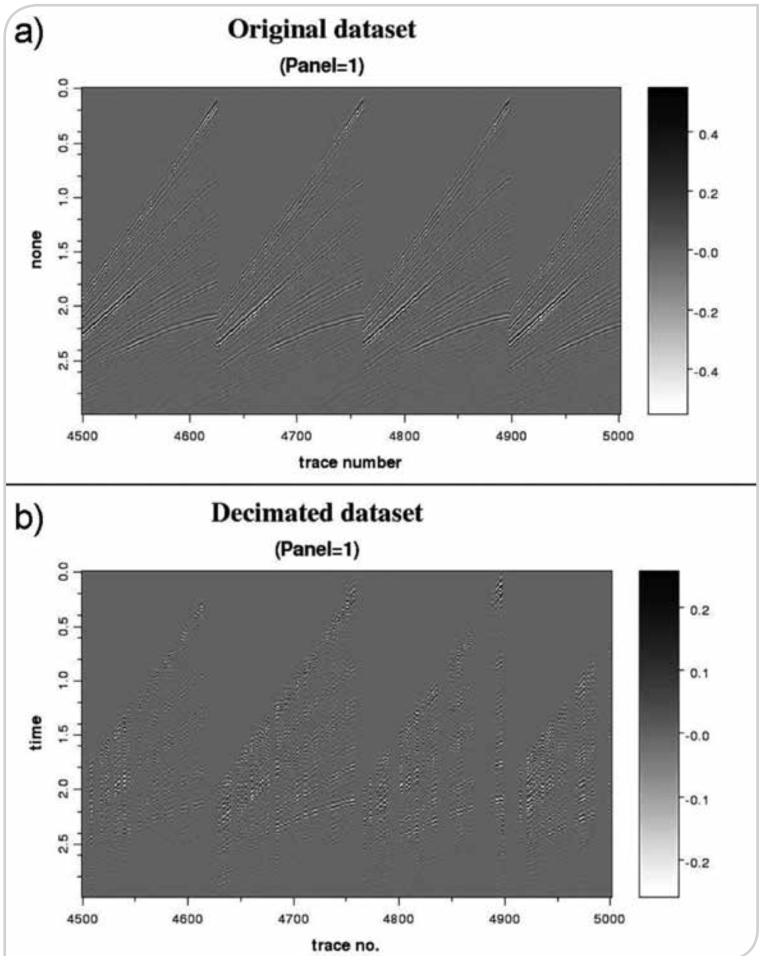

Figure 2. Detail of the Marmousi dataset a) original; b) decimated, after randomly killing 75% of the traces.

Figure 3a shows a common-image gather obtained with conventional migration of the original dataset. Note the aliasing noise which slightly contaminates the gather. For comparison, note the cleaner appearance of the same gather after PLSM, **Figure 3b.**

They exhibit different amplitude scales, but until now benefits are mainly confined to aliasing noise reduction, which is achieved by the preconditioning function. **Figures 4** show the same comparison as **Figures 3**, using the decimated dataset. The conventionally migrated common–image gather exhibits strong aliasing artifacts affecting flat seismic reflections. Such artifacts are highly attenuated by PLSM, and thus seismic events improve their continuity. However, an additional and very important benefit relates to the amplitudes.

Comparing scale bars in **Figures 3a** and **4a** (conventional migration), it is evident that amplitudes highly disagree. Irregularities induced by decimation are responsible for this inconsistency. In contrast, when comparing **Figures 3b** and **4b** (PLSM) shows the amplitude range almost coincident. This amplitude recovery effect was achieved by least-squares migration *sensu stricto*.

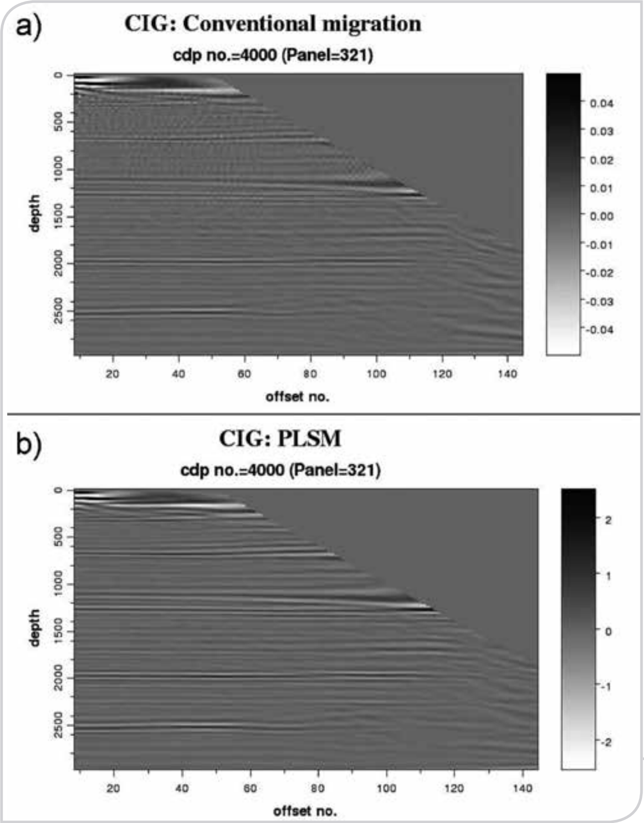

Figure 3. Common image gathers after a) conventional migration; b) PLSM. Original dataset was used.

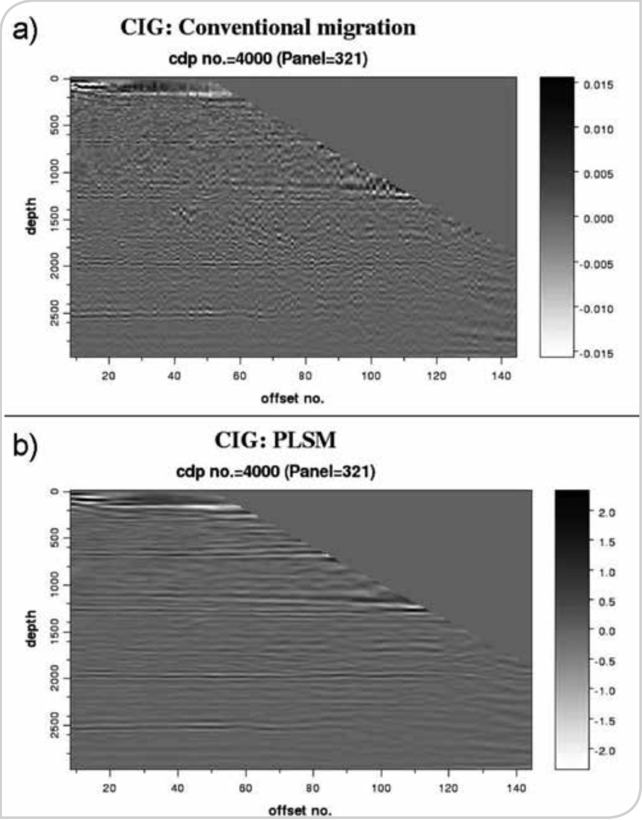

Figure 4. Common image gathers after a) conventional migration; b) PLSM. Decimated dataset was used. Compare to **Figure 3**.

Figure 5a shows a near-offset section obtained with conventional migration of the original dataset. As in the previous comparison, only mild aliasing noise contaminates the image. Such noise is greatly attenuated by PLSM, **Figure 5b**. In **Figures 6** the same comparison with decimated data is shown. Conventional migration, (**Figure 6a**) obtains a very distorted and noisy image without recovering the amplitude range. PLSM, **(Figure 6b)** much better recovers both the image and the amplitude range, as expected. However, a coherent noise indicated by the black arrows arises, present only in the near offsets. Close examination of the input data revealed the presence of low-frequency coherent noise in the data in this sector, which apparently is enhanced by the mean filter when it is undersampled.

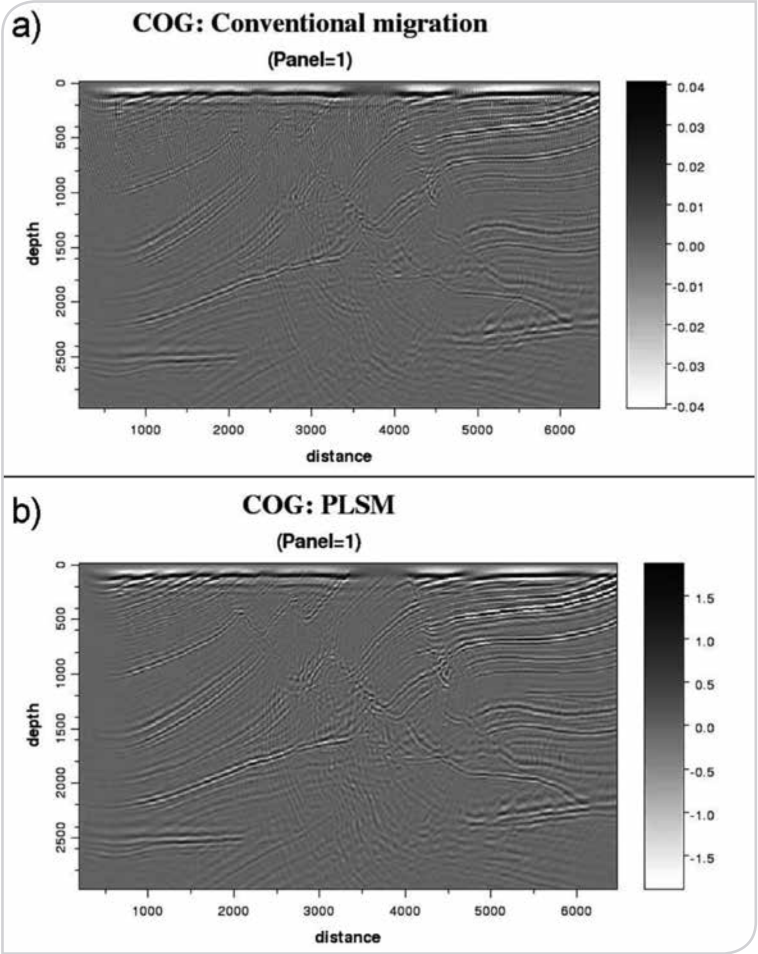

Figure 5. Common offset sections after a) conventional migration; b) PLSM. Original dataset was used.

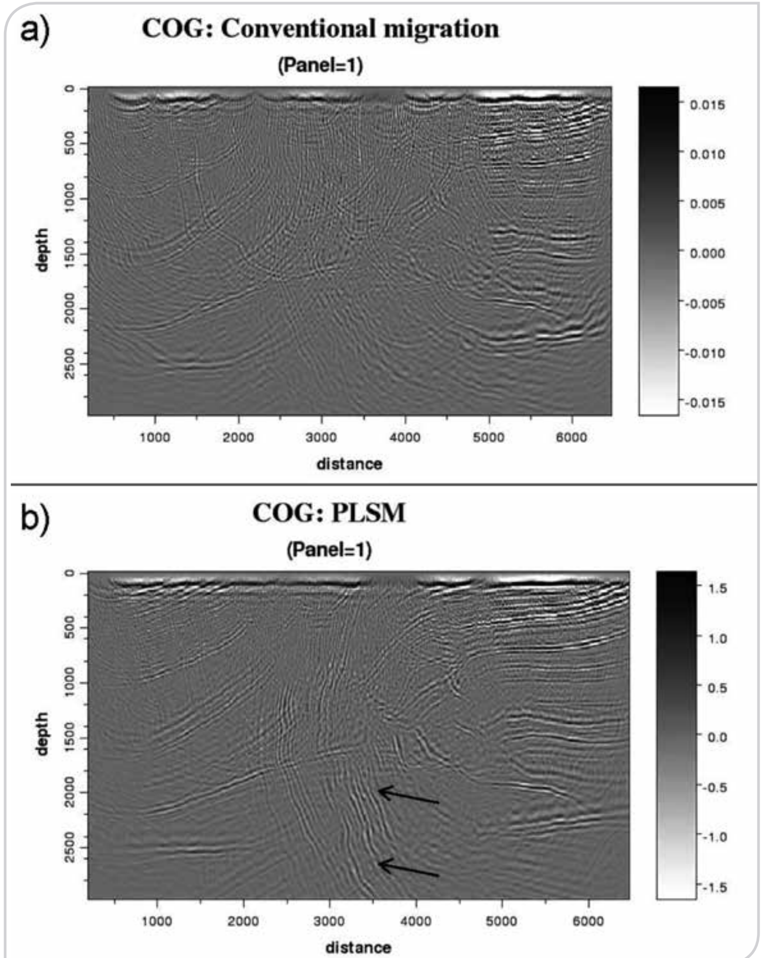

Figure 6. Common offset sections after a) conventional migration; b) PLSM. Decimated dataset was used. Compare to **Figure 5**.

Figures 7 show stack sections of the original data after conventional migration and PLSM, and **Figure 8** shows corresponding sections for the decimated dataset. As expected, aliasing noise is almost completely filtered out by stacking, so the quality of the sections is similar, but PLSM sections from original and decimated datasets have the same amplitude range, as occurred with the gathers. This is again a benefit of leastsquares migration sensu stricto. Closer inspection of **Figure 7**, however, additionally reveals better distribution of amplitudes in the PLSM sections, allowing better visualization of features as shallow faults and cleaner definition of the target area (dashed rectangle). Likewise, the decimated case, **Figure 8** exhibits better continuity and resolution in the shallow reflectors (white lines), although some of the coherent noise observed in the near-offset sections leaks into the PLSM stack section, **Figure 8b**. Usually, we do not expect dramatic changes in image quality of stack sections when comparing conventional migration and least-squares migration, as does occur in 3D data when attenuating acquisition footprint (Cabrales–Vargas, 2011; Guo, 2012).

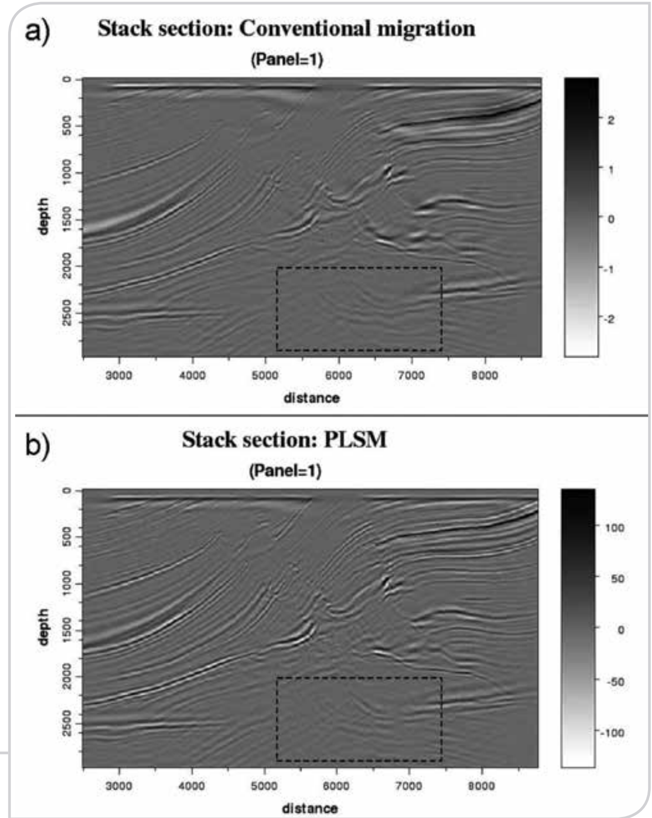

Figure 7. Stack sections after a) conventional migration; b) PLSM. Original dataset was used.

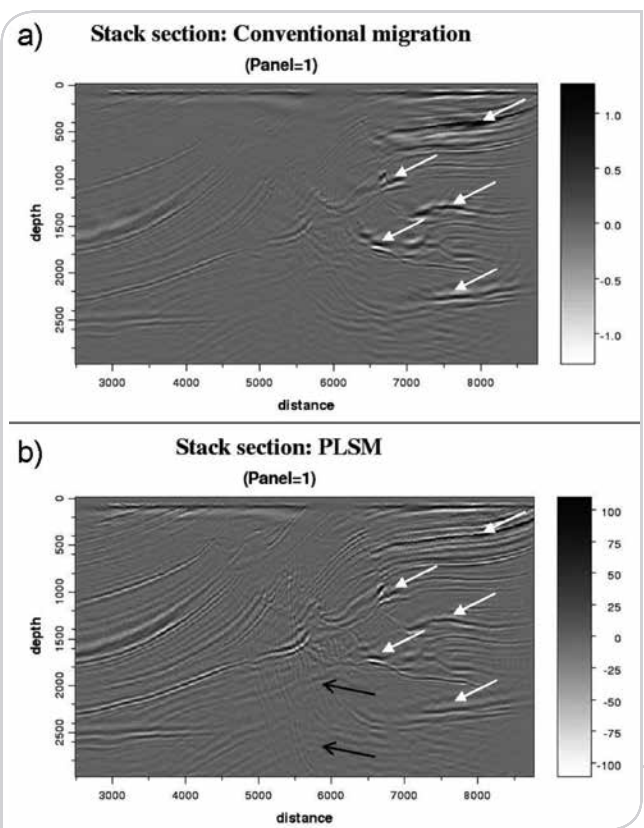

Figure 8. Stack sections after a) conventional migration; b) PLSM. Decimated dataset was used. Compare to **Figure 7**.

Other uses for PLSM

Migrated data with PLSM better represents the relativity of the amplitudes. This leads to especial techniques, other than merely amplitude interpretation. Regularized data can be obtained by demigrating PLSM gathers. In **Figure 9** a comparison of the results of demigrations of the conventionally migrated gathers of **Figure 4a**, and PLSM migrated gathers of **Figure 4b**, is shown both of them corresponding to the decimated dataset. Demigrating conventionally migrated gathers poorly reconstructs the dataset because aliasing artifacts almost remap to the original voids of the decimated dataset. Additionally, the amplitude range significantly disagrees respect to the original dataset. In contrast, demigrating PLSM migrated gathers better regularized the dataset, and the amplitude range almost reached that of the original dataset (compare **Figures 7** to **Figure 2a**). This is because PLSM honors amplitudes better than conventional migration. Another use of PLSM (not analyzed in this work) is diffraction imaging (e.g. Klokov el al., 2010; Asgedom et al., 2012). In such technique, use the residual obtained minimizing the first term of right member of equation (2) to enhance events which do not contribute to reflectivity, e.g., diffraction events related to geological "irregularities" of interest: karsts, fractures, collapse features, etc.

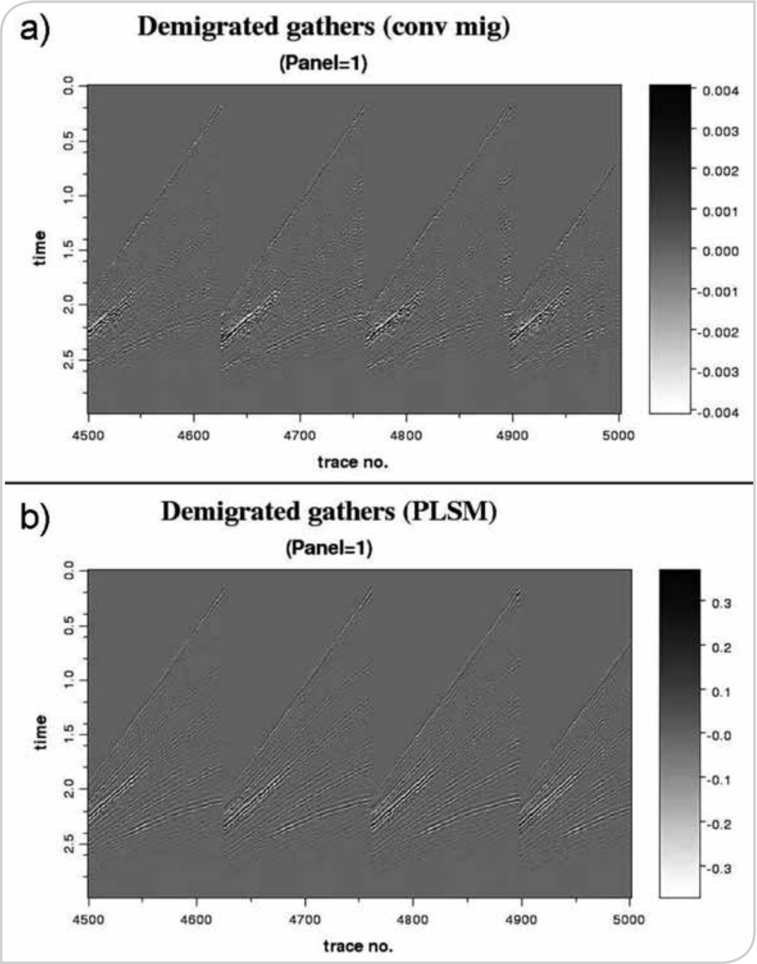

Figure 9. Predicted data from a) conventionally migrated gathers; b) PLSM gathers. Decimated dataset was used. Compare to **Figure 2**.

Conclusions

Preconditioned least–squares migration better preserved seismic amplitudes than conventional migration, attenuating aliasing noise. This constitutes a combined result of minimizing data misfit by least-squares migration sensu stricto and a penalty term. The latter was substituting by a preconditioning function, therefore PLSM could be easily implemented with the conjugate gradient method. In the numerical examples compared conventionally migrated gathers were compared to PLSM migrated gathers in two scenarios: original dataset and decimated dataset. The results suggest good preservation of relative amplitudes in spite of the strong decimation, among a significant reduction of aliasing artifacts. These products can be used as input to regularization of a dataset by demigration, or in more sophisticated techniques like diffraction imaging.

Acknowledgements

I would like to thank to Petróleos Mexicanos and to the Attribute Assisted Seismic Processing and Interpretation (AASPI) consortium. Special thanks to Kurt Marfurt for the comments and corrections on this work, and to Tim Kwiatkowski, who appropriately suggested the term "preconditioned least-squares migration".

References

Alkhalifah, T. 2007. An Anisotropic Marmousi Model. SEP Report 95, Stanford University. http://sepwww.stanford.edu/public/docs/sep95/tariq3/paper_html/.

Aoki, N. y Schuster, G. T. 2009. Fast Least-Squares Migration with a Deblurring Filter. *Geophysics* **74** (6): WCA83-WCA93. http://dx.doi.org/10.1190/1.3155162.

Asgedom, E., Gelius, L. Faccipieri, J., et al. 2012. 2-D Pre- and Post-Stack Diffraction Separation and Imaging. SEG International Exposition and 82nd Annual Meeting, Las Vegas, Nevada, EUA, Vol. 6, 4125-4129. SEG Technical Program Expanded Abstracts 2012.
http://dx.doi.org/10.1190/segam2012-0655.1.

Cabrales-Vargas, A. 2011. Suppression of Aliasing Artifacts on 3D Data Via Constrained Least-Squares Migration. MS thesis, University of Oklahoma. http://geology.ou.edu/aaspi/upload/AASPI_Theses/2011_AASPI_Theses/Alejandro_Cabrales_Vargas_MS_2011_Constrained_least_squares_migration.pdf .

Claerbout, J.F. 1992. *Earth Sounding Analysis: Processing Versus Inversion*. Boston: Blackwell Scientific Publications.

Duquet, B., Marfurt, K.J. y Dellinger, J.A. 2000. Kirchhoff Modeling, Inversion for Reflectivity, and Subsurface Illumination. *Geophysics* **65** (4): 1195-1209.
http://dx.doi.org/10.1190/1.1444812.

Guo, S. 2012. Noise Suppression Using Preconditioned Least-Squares Prestack Time Migration and Imaging of Mississippian Limestone. MS thesis, University of Oklahoma.

Hestenes, M. 1973. Iterative Methods for Solving Linear Equations. *Journal of Optimization Theory and Applications* **11** (4) 323-334. http://dx.doi.org/10.1007/BF00932484.

Hu, J., Schuster, G. y Valasek P. 2001. Poststack Migration Deconvolution: *Geophysics* **66** (3): 939-952.
http://utam.geophys.utah.edu/tomo01/md.pdf.

Klokov, A., Baina, R., Landa, E., et al. 2010. Diffraction Imaging for Fracture Detection: Synthetic Case Study. SEG International Exposition and 80th Ann. Meeting, Denver, Colorado, EUA, octubre 17-22, 3354-3358. SEG Technical Program Expanded Abstracts 2010.
http://dx.doi.org/10.1190/1.3513545.

Lou, M. 2006. Efficient and Accurate Traveltime Computation in 3D TTI Media by the Fast Marching Method. SEG International Exposition and 76th Ann. Meeting, New Orleans, Louisiana, EUA, octubre 1-6. Vol. 1, 184-188. SEG Technical Program Expanded Abstracts 2006.
http://dx.doi.org/10.1190/1.2369885.

Nemeth, T. 1996. Imaging and Filtering by Least-Squares Migration PhD dissertation, University of Utah.

Nemeth, T., Wu, C. y Schuster, G.T. 1999. Least-Squares Migration of Incomplete Reflection Data. *Geophysics* **64** (1): 208-221. http://www-sigproc.eng.cam.ac.uk/~mam59/work/Lit/nemeth-99-1.pdf.

Sethian, J.A. y Popovici, A.M. 1999. 3-D Traveltime Computation Using the Fast Marching Method. *Geophysics* **64** (2): 516-523. http://www.sigmacubed.com/filebin/files/EMI_Journal_Articles/089_geo99sethianpopovici.pdf.

Wang, J. 2005. 3-D Least-Squares Wave-Equation AVP/AVA Migration of Common-Azimuth Data. PhD dissertation, University of Alberta, Edmonton, Alberta. http://saig.physics.ualberta.ca/s/sites/default/files/upload/theses/Juefu.pdf.

Wang, J., Kuehl, H. y Sacchi, M. 2004. High Resolution Least-Squares Wave Equation AVA Imaging: Feasibility Study with a Data Set from the Western Canadian Sedimentary Basin. SEG International Exposition and 74th Ann. Meeting, Denver, Colorado, EUA, octubre 10-15. http://saig.physics.ualberta.ca/s/sites/default/files/upload/conference-proceedings/W_seg04.pdf.

Semblanza de los autores

Ing. Alejandro Cabrales Vargas

Egresado de la Facultad de Ingeniería de la UNAM. Ingresó a Petróleos Mexicanos en agosto de 2002 al proyecto Simojovel del Activo de Exploración Macuspana. De 2004 a 2008 se desempeñó como intérprete sísmico generando y documentando prospectos en el Activo Regional de Exploración Sur. Realiza sus estudios de Maestría en Geofísica en la Universidad de Oklahoma durante el período 2009-2011. Desde octubre de 2011 se desempeña como Supervisor de procesado sismológico en profundidad en la Gerencia de Estudios Geofísicos. Sus principales intereses dentro de la industria del petróleo es el estudio de los métodos de modelado y migración sísmica para la obtención de imágenes del subsuelo, y la interpretación estructural y estratigráfica de dichas imágenes optimizadas. Es miembro de la Society of Exploration Geophysicists (SEG) y de la Asociación Mexicana de Geofísicos de Exploración.

Ing. Alfredo Vázquez Cantú

Egresado de la Escuela Superior de Ingeniería y Arquitectura del Instituto Politécnico Nacional, ingresa a Petróleos Mexicanos en 1990, desarrollando labores como Geofísico analista en la Superintendencia de Operación Geofísica, en la Región Sur. De 2001 a 2003 realizó estudios de Maestría en Geofísica en la Universidad Nacional Autónoma de México. Desde 2004 funge como Supervisor y responsable de procesado sísmico en profundidad. En el período 2011-2012 estudia la Maestría en exploración de hidrocarburos en la Universidad de Aberdeen, Escocia.