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Bertrand Equilibrium for a Pension Fund Management Industry in a market with complete information

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Abstract

There is a little literature regarding the industrial Organization of the Pension Funds Management Industry. In this paper it is proposed that in the Mexican case it could be analyzed using the Bertrand model of duopoly. Although the model makes the unreal assumption that workers have full and perfect information, the results show that, what is relevant in terms of public policy, is increasing the knowledge of the workers about the system rather than promoting the creation of new firms. As long as the workers know that the wealth belongs to them and it is not a tax or paper work, competence will provoke that their wealth increase.

Key words: Pension funds management firms, Bertrand competence, Herfindhal index.

Resumen

Existe poca literatura acerca de la organización industrial de los Fondos de Ahorro para el Retiro. En este texto se propone que,

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en el caso de México, ésta podría analizarse a la luz del análisis de duopolio de Bertrand. Aunque el modelo parte del supuesto poco realista de que los trabajadores tienen información perfecta y completa, los resultados indican que, lo realmente relevante en términos de política pública, es promover la información hacia los trabajadores más que fomentar la creación de nuevas Afores. En la medida que los trabajadores sepan que son dueños de su riqueza y no vean a las Afores como un impuesto o un trámite, la competencia provocará que su riqueza total aumente.

Palabras clave: Administradoras de fondos para el retiro, competencia estilo Bertrand, índice de Herfindhal.

Introduction

The wave of privatization of pension systems all over the world gave rise to a new industry: the management of pension funds. Although there are several papers and even books about the macroeconomic effects of private pension systems a little attention, if any, was been given to the industrial organization aspects of firms that compete to manage worker's funds.

Given the nature of the industry, we will claim that the competition among Afores is *à la* Bertrand. There are a few empirical cases of competition that can be analyzed under this theoretical framework. Since the private pension management is a new industry, there is a little literature regarding this issue. In fact, most of the models assume implicitly or explicitly, that the competition in this industry is a kind of Cournot competition.

Actually, many of the most important books about pensions do not deal with the subject of the industrial organization of the firms. Valdes-Prieto (1997) does not include any discussion regarding this subject in his pioneer book. Maybe this is due that when the book was edited, many countries where still in the discussion of the pension reform.

Scheil-Adlung (2001) focuses, mainly in the welfare effects of the reform of the Social Security privatization, but she does not say anything about the industrial organization of the firms.

Blake (2006) is an excellent reference for the economics of pensions. His book has several approaches to it. Unfortunately, there is something missing: a chapter about the way the pension funds managing firms compete.

The paper of Beristain and Espindola (2001) is a good example of an attempt to develop a model of competition of the Afores under the Cournot framework. García Huitron and Rodríquez Gómez (2002) is another good example of an attempt in the same path. The main problem with the approach of the above authors is that they assume competition in quantities not in prices.

Impavido, Lasagabaster and García-Huitrón (2010), states that in Latin America countries exist a power market of the pension fund manager firms. In any case their work is a general approach of several developing countries, not specifically in México. Nevertheless it is one of the only works that deals with this important issue.

One important feature of the new pension system in Mexico is that the firms basically compete in prices rather than in quantities. Actually, the current regulation of the industry does not allow a single firm to have more than 19% of the potential market. So even if they compete in quantities, these are limited by law. Another competition form is given by the return of the funds that the Afores offer. But also in this case, the law is very restrictive, so there are no many financial instruments in which they can invest the funds. This is why it is plausible to think in a competition *à la* Bertrand, rather than of the Cournot type. Impavido et al (Op Cit), will probably disagree, but in this paper we will show that there is empirical evidence that supports this statement.

One example of the Bertrand approach to the competition of the pension management firm's competition is due to Ferro (2002), who develops a theoretical frame to analyze the Argentine Pension Funds industry. In his dissertation, Ferro states that in this industry the competition is given by prices, so the "right" approach should be the Bertrand style competition. In the first stage he assumes homogeneous product and then he makes a differentiation on it but, in any case, the Bertrand scheme is always presented. (Ferro, 2002:33-42)

One of the problems when dealing with the pension funds management industry is the knowledge of the workers about the pension system. Fobtonogov and Murthi (2005) made a comparative study of the fees and costs of the managing the pension accounts in four countries: Croatia, Hungary, Kazakhastan and Poland, and they discover that the workers poorly understand the fee structure of the firms. Although we do not know of the existence of any survey about the knowledge of Mexican workers, it seems that they face the same problem: there is a difficulty in understanding the fee structure of the Afores.

This is a big problem when we face the model against the reality: In real life workers do not have neither complete nor perfect information. This implies that it is difficult to apply any theoretical model. However, this problem is shared for any competition model. So we will focus in one theoretical frame and we will show that there are elements that suggest that soon the industry could reach a Bertrand Equilibrium.

The regulation issue

Some papers that talk about pension regulation do not say anything about the kind of competition that this industry has. A good example of this is Vittas, (1998) who talks about the regulatory Controversies of Private Pension Funds, but he talks about taxes, operational controls, investment limits, etcetera, but does not talk about the market structure. In a way one could say that he is assuming a Cournot competence in an industry where the firms could have some monopoly power. This fact, again, is stated by Impavido, et al (Loc. cit).

Another paper that deals with the regulation issue is the one made by the economic studies management office of the Indecopi of Peru (2002). Their approach is an international comparison of the fees that the pension funds management firms charge in several Latin American Countries. They compare the returns, the fee and the regulation, but their study does not have any theoretical approach regarding the competition of the industry.

In this paper we will describe the new industry of pension funds management in the case of Mexico. In the first section it will describe the industry, its rules and the ways that can be charged to workers for managing their funds. The second section describes organizational aspects this industry is likely to have; the third one puts forward a version of Bertrand model for the industry. The fourth section relaxes the assumptions of the previous model by allowing variable cost and random return. The fifth section shows a dynamic model to find out the best strategy for the workers. Last section concludes.

1. The Mexican new market

Chile was the pioneer country in privatizing parts of the social security system. The privatization put private firms in charge of management of workers' pension funds. Several countries followed the steps of Chile. Mexico privatized its system too, and in July 1997 a new industry was born, the industry of Afores (Administradoras de Fondos para el Retiro, pension funds managers). The service, these firms sell to the workers is, of course, managing their pension funds. Compare with other financial instruments available for the workers, they offer high return on workers' savings expecting to have enough wealth in the moment they become pensioners.

The system is simple: every worker has to contribute a percentage of his salary to the pension funds; this amount is supplemented by contributions from both the firm that employs him and the government. This means that the worker's saving has three components: his own saving plus the firm and the government contribution. The sum of the contributions is at least 11.5% of the worker's wage; the government contribution is a fixed percentage of the minimum wage of July 1997, so the higher the wage, the lower the total savings of the worker (as a percentage of the wage), in any case, the least the worker saves is still 11.5% of his wage.

Another feature of the system is that the savings have two main goals: creating a wealth for housing and creating a wealth for pensions. That is why 5% of the saving is managed by an institution in charge of building houses for the workers; the remaining (6.5%) is managed by the Afores. In any case, if the worker does not buy a house using his savings, he can request them when the retirement time arrives. This paper will only focus on the Afores, that is, we will take out the 5% that is managed by the housing manager.

When the system started, the Afores had three possible ways of charging a fee to the workers: 1) charging a fee as a percentage of the worker's wage, 2) charging a percentage of the savings balance at the end of the year and 3) taking a part of the net return over the savings. They all have an equivalency formula. In this paper we develop the framework for the three fees, but will focus on the first and, particularly, the second fees.

The new industry is young. When the retirement moment arrives, the worker has to choose between requesting his wealth or letting the Afore manage his wealth so the firm will pay to the worker a pension for the rest of his life. But this stage is still far away, so we will focus exclusively in the first stage, meaning the saving process.

Before dealing with the model we will develop, it is convenient to take into account some information regarding this market. First of all, for many workers the Afore is nothing but a requirement to get a job; for the firm it is nothing but a tax on wages; this means that in several cases they do not see the savings as an asset, but as a liability; maybe it will be clear what they are when workers start receiving pensions from these funds. In a way this means that there is not full information in the market, not only regarding weather it is an asset or liability, but also regarding which Afore is the best for each worker. Second, time is important since we are dealing with long term saving. In the next section of this paper, we will assume that both firms and workers have complete information about the market and they optimize giving this information and that workers and firms maximize every unit of time independently of the others. After this, we will relax the assumptions of the first model to allow variable cost for the firms and to include uncertainty. Although the assumptions for those models are not very realistic, it is a feature that many economic models assume and it could be the first step toward a model with intertemporal approach, which we will develop in the last section of this paper.

2. Firms and costumer behavior

This section analyzes the behavior of both firms and workers behavior; in the next section we will put them together to reach the Bertrand equilibrium.

2.1 Firms behavior

It is more likely that firms will have more information than consumers. In any case firms want to maximize their profits. We are dealing with a homogeneous service, which is the management of the workers savings, where the number of accounts they can manage is limited. This means that the competition between firms will be given by fees they receive from workers. In other words the competition will be in terms of fees rather than quantities.

This feature of the market lead us to Bertrand model of competition, the difference is that in the classical Bertrand model the firms compete just in terms of one single price while in this case there are several prices[2]. Actually from March 15th, 2008[3], the fee will be only over balance, in this model we will see the implications of this new rule for both, workers and firms. We will see below that the Bertrand paradox still holds (see Tirole, 1998, pp. 209, 10) and that firms will charge a fee that just allows them to recover their cost.

2.2 Consumers behavior

As I stated earlier, most of the workers do not know their savings are managed by the Afores. They are free to choose the Afore they want, but in many cases they do it because Afores' agents are after them and because it is a requirement to work in the formal sector of the economy. Nevertheless in this section of the paper I will assume that all workers have full information and that their objective is to maximize their savings every unit of time in a way that, given that we are dealing with a homogeneous service, they choose the Afore that gives them the highest net return.

In the traditional Bertrand model when the firms sell a homogeneous product and compete in prices, the solution is that the price has to be equal to de marginal cost, reaching the competitive solution (Tirole, 1998). In that case, each firm has half of the market. This is not a crucial assumption in the Bertrand model neither will be in the model we will develop. In any case consumers will be indifferent between firms when they pay the same net return and they will choose either.

[2] For a case where firms compete in a differentiated product, see Gibbons, 1992 pp. 21,2.
[3] See Diario Oficial de la Federación 15 de junio, 2007. "Decreto por el que se reforman y adicionan diversos artículos de la Ley de los Sistemas de Ahorro para el Retiro", art. 37.

3. Bertrand Model with competition in three fees

After knowing the main features of the firms and consumers, we can model their behavior together. In the first step we will assume the existence of monopoly and we will show how the monopolist extracts every cent of the worker. Then we will jump to the duopoly case and we will show that both firms have to charge exactly the same fees and that they reach the competitive solution.

In this first stage, we will assume that the workers try to maximize the maximum return over their net saving period by period. This assumption is realistic since in real life they can change the Afore that has their account for another with lower fee of higher return if they want to. We will assume that every period they save one unit and try to maximize this unit plus the net return.

3.1 The general problem under current law

As we stated before, the law allows the Afores to charge three kinds of fees: under saving flows, under return and over balance. Let us take the saving as a *numerarie,* in a way that every time the worker saves one unit. In those terms, workers problem is:

$$\max\ \pi = (1-f)[1+r(1-\rho)](1-s) \qquad (1)$$

while firms want to

$$\max\ \pi = f + r\rho + (1-f)(1+r(1-\rho))s - c \qquad (2)$$

$$0 \le f \le 1$$
$$0 \le \rho \le 1$$
$$0 \le s \le 1$$
$$r > 0$$
$$c > 0 \qquad (3)$$

where

f: fee firms charge the worker by managing his pension fund and is a percentage of the flow the worker saves every unit of time.

r: return over savings the firms pays to the worker. In this step I will assume that it is exogenously given by the market, that it is the same for every firm and that the firm does not have any power over it. Later we will allow be a random variable.

s: fee Afores charge over balance.

ρ: fee over return, it is a charge made exclusively over the return of the worker savings.

c: cost of managing the saving accounts of the workers, I will assume that is a fixed cost per worker. Later we will relax this assumption to allow it to be random.

3.2 Fees under monopoly

If we have a market with one single firm offering the service of managing the workers pension funds, the firm will take advantage of its power to extract every cent of the worker.

The general case

The monopoly faces the next problem:

Max $\pi = f + r\rho + (1-f)(1+r(1-\rho))s - c$ (2)

s. t. $\pi = (1-f)[1+r(1-\rho)](1-s) \geq 0$ (4)

This means that the costumer has to receive at least zero net saving.

It is not difficult to verify that the solution for the monopolist is

$f = 1-\delta, \quad \delta \to 0, \quad 0 \leq \delta \leq 1$, (5)

$\rho = 1-\partial, \quad \partial \to 0, \quad 0 \leq \partial \leq 1$, and (6)

$$s = 1 - \varepsilon, \quad \varepsilon \to 0, \quad 0 \leq \varepsilon \leq 1 \tag{7}$$

Inserting this solution into consumer equation, his problem becomes:

$$\max \ \delta\varepsilon(1 + r\partial), \tag{8}$$

which could be seen as an utility function.

From here, we can see that the consumer will be in his best when the δ, ε *and* ∂ reach their maximum value that is equal to one.

For the monopolist, the profit function, for every account he is managing, becomes:

$$\text{Max} \quad \pi = (1 - \delta) + r(1 - \partial) + \delta(1 + r\partial)(1 - \varepsilon) - c \tag{9}$$

then for the monopoly case, the solution is

$$\delta = \varepsilon = \partial = 0, \tag{10}$$

then $\pi = 1 + r - c$, and the consumer gets zero. In other words, the monopoly gets the saving plus the return minus the cost.

A more realistic (and simpler) case

In practice, the Afores were charging only under two concepts: flow and balance, which means that $\partial = 1$. This means that the consumer problem becomes:

$$\max \ \delta\varepsilon(1 + r) \tag{11}$$

and the firm problem is now:

$$\max \ \pi = (1 - \delta) + \delta(1 + r)(1 - \varepsilon) - c \tag{12}$$

The result is essentially the same:

$$\delta = \varepsilon = 0, \qquad (13)$$

And again, the worker gets zero. In next section we will develop the framework for the duopoly case under the general case and under two fees: flow and balance. This last is a more realistic exercise, since in practice is what the Afores were doing during the first years of their existence.

The case with the new rules

I have stated before that starting March 2008, the fee allowed will be only over flow. If that is the case, this implies that $\delta = 1,$ so the consumer problem becomes:

$$\text{Max } \varepsilon(1+r) \qquad (11^*)$$

and the firm problem is now:

$$\text{max } \pi = (1+r)(1-\varepsilon) - c \qquad (12^*)$$

The solution is:

$$\varepsilon = 0, \qquad (13^*)$$

Again, the monopolist gets everything and the consumer gets zero.

3.3 Duopoly case

The general case

Under the general case, each firm faces the next profit function:

$$\pi_i = (1-\delta_i) + r(1-\partial_i) + \delta_i(1+r)(1-\varepsilon_i) - c_i \quad i = 1,2 \qquad (14)$$

From (8 it is clear that if $\delta_1\varepsilon_1(1+r\partial_1) > \delta_2\varepsilon_2(1+r\partial_2)$, then firm 1 gets the whole market and firm 2 leaves. Since the converse is also true, this means that in order to share the market it has to be true that

$$\delta_1\varepsilon_1(1+r\partial_1) = \delta_2\varepsilon_2(1+r\partial_2) \qquad (15)$$

This is a kind of Bertrand game, so the firms will compete in prices up to the level where profits will be zero. This means that

$$\pi_i = (1-\delta_i) + r(1-\partial_i) + \delta_i(1+r)(1-\varepsilon_i) - c_i = 0, \quad i = 1,2 \qquad (16)$$

From this last equation, it is easy to see that

$$\varepsilon = \frac{1+r+r\partial(\delta-1)-c}{\delta(1+r\partial)} \qquad (17)$$

As long as we keep this identity the profit will be equal to zero. Since in real world the Afores are charging for different fees, let us find out the relationship between them. From equation (8 we can see that a maximum for the consumer is reached when $\varepsilon = \delta = 1,\ and\ \partial = 1 - \frac{c}{r}$. Let us see what happens when one or two of the parameters are equal to one.

Case 1:

$$\partial = 1, \quad \Rightarrow \quad \varepsilon = \frac{1+r\delta-c}{\delta(1+r)} \qquad (17.1)$$

Case 2:

$$\delta = 1, \quad \Rightarrow \quad \varepsilon = \frac{1+r-c}{1+r\partial} \qquad (17.2)$$

Case 3:

$$\partial = \delta = 1, \quad \Rightarrow \quad \varepsilon = \frac{1+r-c}{1+r} \qquad (17.3)$$

Case 4:

$$\varepsilon = 1, \quad \Rightarrow \quad \delta = 1+r-r\partial-c \qquad (17.4)$$

Case 5:

$$\varepsilon = \delta = 1, \quad \Rightarrow \quad \partial = \frac{r-c}{r} \tag{17.5}$$

Case 6:

$$\partial = \varepsilon = 1, \quad \Rightarrow \quad \delta = 1 - c \tag{17.6}$$

In the utility function the simplest case is 5. It is easy from (17.5) to verify that

$$\varepsilon = \delta = 1, \; and \; \partial = 1 - \frac{c}{r} \tag{17.5}$$

Here the worker receives 1+*r*-*c* and the firms receive zero. This means that the worker receives full return less the cost of managing his account. Case 1 was the more common up to March 2008, but under the new rules, starting that date, case 3 is the one that holds.

So far, it seems clear the optimal fee for the worker is just under return, at least at the early stages of saving. Now let us see what happens when the fee under return is equal to zero (the status quo before March, 2008).

The status quo before March 2008

This is case 1, (17.1), here each firm faces the next profit function:

$$\pi_i = (1-\delta_i) + \delta_i(1+r)(1-\varepsilon_i) - c_i \tag{18}$$

Again $\delta_1\varepsilon_1(1+r) > \delta_2\varepsilon_2(1+r)$, then firm 1 gets the whole market and firm 2 leaves. Here also has to be true that in order to share the market we need that

$$\delta_1\varepsilon_1(1+r) = \delta_2\varepsilon_2(1+r) \neq 0 \tag{19}$$

One more time we have a kind of Bertrand competition, so the firms will compete in prices up to the level where profits will be zero. This means that

$$\pi_i = (1-\delta_i) + \delta_i(1+r)(1-\varepsilon_i) - c_i = 0, \quad i = 1,2 \qquad (20)$$

From (20), it has to be true that for each firm, since $\partial = 1$, we have case 1. From (17.1) we can see that

$$\delta = \frac{1-c}{\varepsilon(1+r)-r} \qquad (21)$$

Given the condition (19, this means that neither of the variables (δ, ε) can be equal to zero. So again we have two extremes, when the delta and epsilon are equal to one. Let us start with the March, 2008 status quo.

If $\delta = 1$, $\varepsilon = \dfrac{1+r-c}{1+r}$, in this case the worker receives 1+*r*-*c* and the firms receive zero. In other words, the worker receives full return less the cost of managing his account.

If $\varepsilon = 1$, $\delta = 1-c$ (case 6), the worker receives 1+*r*-*c*-*rc* and the firms get zero. This means that the worker receives the return less the cost times the interest rate. This happens because when $\delta = 1$, in this case, the fee is charged at the end of the period, and if $\varepsilon = 1$, at the beginning, so the fee is higher.

It is important to remark that under the status quo case, and in the early stages of saving, the optimal fee for the consumer is over balance. This is because he gets the return over his full saving while when the fee is under the flow, the interest is generated under a lower base. In both cases the firms get zero revenue, but when the fee is under the balance, the worker is better off. Let us see what happens when any of the fees are zero.

From (17.1 we know that:

$$\varepsilon = \frac{1+\delta r - c}{\delta(1+r)} \tag{22}$$

Plugging (22) in the utility function (11), we get:

$$U = 1 + \delta r - c \text{ and } \pi = 0 \tag{23}$$

This means that the higher the fee under flow, the lower the utility of the consumer, even when the firms get zero revenue. It is easy to see that the consumer reaches his maximum when $\delta = 1$ meaning, when the fee under flow is zero, perhaps this is the idea behind the new rules that set $\partial = \delta = 1$, this means that the charge will be only over balance.

The real world does not allow the consumer to have full and complete information. That could be one of the reasons why the structure of fees does not conduce toward a perfect competition solution. Nevertheless, we will see in table one (section 5.4) that in the last months the fees have changed so now most of the Afores charge a fee under balance. In section 5 we will see that in a dynamic approach, it could be convenient for the worker having a menu of Afores that charge over any of the three kinds of fees. But first let us relax the initial assumptions.

4. Relaxing the assumptions: variable cost and random return

In the previous case we assumed that the cost of managing the fund was constant and equal to *c*. We also assumed that the return was fixed, given by the market and that the Afore do not have any power over it. In this section we will broke those assumptions, the first part will focus in the cost, next part will assume that the firm can choose financial instruments to increase the return, but this has a cost. The essential results still holds, but now it will be clear that in the presence of uncertainty and variable cost the firm will charge a higher fee.

4.1 Variable cost

Afores agents always said that their Afore is the one with lower fees, with higher returns or both. They can invest the worker wealth in several financial assets. Under current law many of those assets have to be risk free, but there is room to look for those with higher return. Targeting the best financial instruments implies a cost for the Afore, so the function cost we propose is:

$$c(r) = c + br^{\alpha}$$

where

c is the fix cost of managing the account.
b *and* α are a parameter given by the technology, in this paper we assume that
$b \geq 0$ and that
$\alpha > 1$

This last condition only implies that the higher the return, the higher the cost. Further development of this model will show that under uncertainty, the fees will be higher.

Monopoly case

In this situation the outcome of the game is essentially the same: the monopoly gets everything and the worker gets zero. The only difference is that the cost now has different face.

Again, worker's objective is

$$\max\ (1-f)[(1+r(1-\rho)](1-s) \qquad (1)$$

while firms want to

$$\max\ \pi = f + r\rho + (1-f)(1+r(1-\rho))s - c - br^{\alpha} \qquad (2')$$

The solution implies that the consumer gets every penny and consumer gets zero. So the solution is:

$$\pi = 1 + r - c - br^{\alpha}$$

and consumer gets zero.

Duopoly case

Again the competence in prices reduces the profits up to the level where they are equal to zero. The only difference between this case and the one showed in section 3.3 is the cost. The solution is pretty similar.

Under the general case, each firm faces the next profit function:

$$\pi_i = (1 - \delta_i) + r(1 - \partial_i) + \delta_i(1 + r)(1 - \varepsilon_i) - c_i - b_i r^{\alpha} \qquad (14')$$

Following the same approach than in section 3.1, it is easy to verify that for each firm it has to be true that

$$\varepsilon = \frac{1 + r + r(\delta - 1) - c - br^{\alpha}}{\delta(1 + r)} \qquad (17')$$

As long as we keep this identity the profit will be equal to zero. Let us see the same six cases:

Case 1:

$$\partial' = 1, \quad \Rightarrow \quad \varepsilon' = \frac{1 + r + r(\delta' - 1) - c - br^{\alpha}}{\delta'(1 + r)} \qquad (17.1')$$

Case 2:

$$\delta' = 1, \quad \Rightarrow \quad \varepsilon' = \frac{1 + r - c - br^{\alpha}}{1 + r\partial'} \qquad (17.2')$$

Case 3:

$$\partial' = \delta' = 1, \quad \Rightarrow \quad \varepsilon' = \frac{1 + r - c - br^{\alpha}}{1 + r} \qquad (17.3')$$

Case 4:

$$\varepsilon' = 1, \quad \Rightarrow \quad \delta' = 1 + r - r\partial' - c - br^{\alpha} \qquad (17.4')$$

Case 5:

$$\varepsilon' = \delta' = 1, \quad \Rightarrow \quad \partial' = \frac{r - c - br^{\alpha}}{r} \qquad (17.5')$$

Case 6:

$$\partial' = \varepsilon' = 1, \quad \Rightarrow \quad \delta' = 1 - c - br^{\alpha} \qquad (17.6')$$

Looking only at the workers side, it is easy to see that they reach their maximum in the simplest case: from (17.5' can verify that

$$\varepsilon' = \delta' = 1, \; and \; \partial' = 1 - \frac{c + br^{\alpha}}{r} \qquad (17.5')$$

Here the worker receives $1 - c - br^{\alpha}$ and the firms receive zero. This means that the worker receives full return less the cost of managing his account. The only difference here is that the cost is not longer fix. This could imply that the fees would be higher. In this stage there is not a really big difference, but it will be when we include uncertainty in the interest rate. That is next step.

4.2 Random return

We have stated before that the Afores have a small number of assets in which they can invest. Although the number is increasing, it is still too short. Nevertheless it is plausible to think that in a near future the number of assets in which will be possible to invest will be higher.

Since all Afores have the same rules, we will assume that the expected return is the same for every firm. We also assume that the technology they use is the same, so the cost function is equal for all of them.

Let us remember that

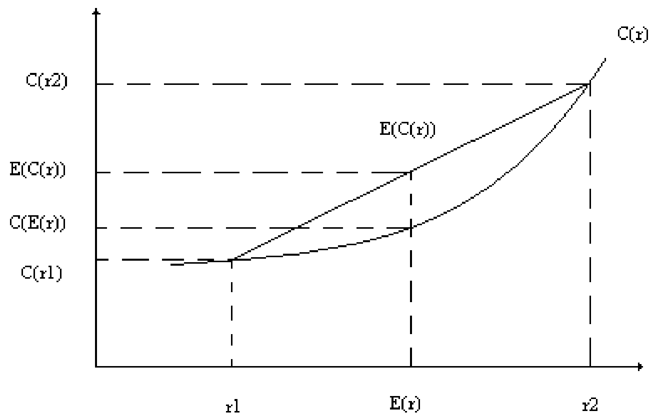
$c(r) = c + br^{\alpha}$, where
$b \geq 0$ and $\alpha > 1$

Given those conditions, the expected cost will be:

$$E(c(r)) = E(c + br^{\alpha}) = c + bE(r^{\alpha})$$

Since $\alpha > 1$, and therefore *c(r)* is a convex function, it is clear that $E(r^{\alpha}) > E^{\alpha}(r)$, which implies that now the cost is higher than in the case where there was not uncertainty (see chart 1)[4].

Chart 1. Cost of expected return and expected cost



Now we can return to our analysis of monopoly and duopoly.

Monopoly case

Worker's objective is

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[4] As an example, let us remember that in probability theory they teach that: $V(x) = E(x^2) - E^2(x)$, which yields, $E(x^2) = V(x) + E^2(x) \geq E^2(x)$. In this case, it correspond to the case where $\alpha = 2$.

max $(1-f)+E(r)\rho+(1-f)(1+E(r))(1-\rho))s$ (1)

while firms want to

max $E(\pi)=f+E(r)\rho+(1-f)(1+E(r))(1-\rho))s-c-bE(r^{\alpha})$ (2'')

Again the monopoly extracts every cent, so the solution is:

$$E(\pi)=1+r-c-bE(r^{\alpha})$$

and consumer gets zero.

Duopoly case

The difference now is that we are including uncertainty, under the general case each firm faces the next expected profit function:

$$E(\pi_i)=(1-\delta_i)+E(r)(1-\partial_i)+\delta_i(1+E(r))(1-\varepsilon_i)-c_i-b_iE(r^{\alpha}) \qquad (14'')$$

Following the same approach than in section 3.1, it is easy to verify that for each firm it has to be true that

$$\varepsilon=\frac{1+E(r)+E(r)\partial(\delta-1)-c-bE(r^{\alpha})}{\delta(1+E(r)\partial)} \qquad (17'')$$

As long as we keep this identity the profit will be equal to zero. Since $E(r^{\alpha})>E^{\alpha}(r)$, we can see immediately that the fee over balance will be higher than when there is not uncertainty.

Let us see the same six cases:

Case 1:

$$\partial''=1, \quad \Rightarrow \quad \varepsilon''=\frac{1+E(r)+E(r)(\delta-1)-c-bE(r^{\alpha})}{\delta''(1+E(r))}<\varepsilon' \qquad (17.1'')$$

Case 2:

$$\delta'' = 1, \quad \Rightarrow \quad \varepsilon'' = \frac{1 + E(r) - c - bE(r^{\alpha})}{1 + E(r)\partial} < \varepsilon' \qquad (17.2'')$$

Case 3:

$$\partial'' = \delta'' = 1, \quad \Rightarrow \quad \varepsilon'' = \frac{1 + E(r) - c - bE(r^{\alpha})}{1 + E(r)} < \varepsilon' \qquad (17.3'')$$

Case 4:

$$\varepsilon'' = 1, \quad \Rightarrow \quad \delta'' = 1 + E(r) - E(r)\partial'' - c - bE(r^{\alpha}) < \delta' \qquad (17.4'')$$

Case 5:

$$\varepsilon'' = \delta'' = 1, \quad \Rightarrow \quad \partial'' = \frac{E(r) - c - be(r^{\alpha})}{E(r)} < \partial' \qquad (17.5'')$$

Case 6:

$$\partial'' = \varepsilon'' = 1, \quad \Rightarrow \quad \delta'' = 1 - c - bE(r^{\alpha}) < \delta' \qquad (17.6'')$$

In the utility function the simplest case is case 5. It is easy from (17.5) to verify that

$$\varepsilon'' = \delta'' = 1, \; and \; \partial'' = 1 - \frac{c + bE(r^{\alpha})}{E(r)} < \partial' \qquad (17.5'')$$

Since

$$\partial'' < \partial'; \quad \varepsilon'' < \varepsilon' \quad and \quad \delta'' < \delta' \quad \Rightarrow \quad f'' > f', \quad \rho'' > \rho' \quad and \quad s'' > s'$$

This implies that under uncertainty the fees charged to the worker are higher than when there is not. There is not a big difference between this case and the previous, but now we can clearly see that when the interest rate is random, the fees are higher. In other words the uncertainty has a cost and it is paid by the worker.

So far, the model developed here is static, meaning that is one shot game. In real life, workers have to save at least 25 years, and the saving process takes place every month, this implies that a better analysis could be done by allowing maximizing over time. Although a full development of a dynamic model is beyond the scope of this paper, in next section we will put some insights regarding this topic.

5. A dynamic approach: finding the optimal path for the worker

In previous sections we have assumed that the saving process takes place one single time. This is not true in real life, so in this section we will develop a framework to analyze the saving process in a dynamic fashion. In the first stage we will assume that the firm charges the three fees, next we will see the six cases developed before and we will remark the current status quo as well as the one will be starting March 2008 to find out what is the best strategy for the worker.

5.1 Finding the wealth of the worker at time *t*

Since the saving process is dynamic, it is convenient to find out a formula to know the wealth of the worker in any time. Let us keep in mind that we are dealing with unit of saving, so every period, there will be some *net saving (NS)*:

$$NS = (1-f)[(1+r(1-\rho)](1-s)$$

taking (5), (6) and (7) into account,

$$NS = \delta\varepsilon(1+r\partial)$$

The wealth at any period of time will be the previous wealth plus net return plus the net saving of that period, in other words:

$$W_t = W_{t-1}\varepsilon(1+r\partial) + NS = W_{t-1}\varepsilon(1+r\partial) + \delta\varepsilon(1+r\partial) = (W_{t-1}+\delta)(1+r\partial)\varepsilon \quad (24)$$

From (24), it is also true that

$$W_{t-1} = W_{t-2}\varepsilon(1+r\partial) + NS \quad (25)$$

$$W_{t-2} = W_{t-3}\varepsilon(1+r\partial) + NS \qquad (26)$$

Replacing (24) into (25) and (26) and after a backwards iterative process, we can find that

$$W_t = \varepsilon(1+r\partial)(\varepsilon(1+r\partial) + \varepsilon^2(1+r\partial)^2 + ... + \varepsilon^{t-1}(1+r\partial)^{t-1}) \text{ .... } (27)$$

In other terms:

$$W_t = \delta\varepsilon(1+r\partial)(\frac{1-(1+r\partial)^t\varepsilon^t}{1-(1+r\partial)\varepsilon}) \qquad (28)$$

Equivalently:

$$W_t = \delta\varepsilon(1+r\partial)(\frac{(1+r\partial)^t\varepsilon^t - 1}{(1+r\partial)\varepsilon - 1}) \qquad (28')$$

From this last equation we can see that the charge over flow (δ) has an important impact in the early stages of saving, while the charges over balance and flow are more important in the latest. Let us see what happens when the fee is only over one fee, so we will have three cases: over flow, over balance and over return.

Case 1: Charging only over flow

In this case, $\varepsilon = \partial = 1,$ so (28') becomes:

$$W_t = \delta(1+r)\frac{(1+r)^t - 1}{r} \qquad (29)$$

Under the assumption of duopoly competition, and assuming that from a dynamic point of view the profits are still zero, and taking into account (17.6), (29) becomes:

$$W_t = (1-c)(1+r)\frac{(1+r)^t - 1}{r} \qquad (30)$$

So when t=1, we are in case 6 of section 3.3. Meaning that the firm has zero profits and the worker gets 1+*r*-*c*-*rc*. Let us see what happens when the fee is under real return.

Case 2: fee over real return

Now $\delta = \varepsilon = 1,$ so (28') becomes

$$W_t^* = (1 + r\partial)\frac{(1 + r\partial)^t - 1}{r\partial} \qquad (31)$$

Assuming zero profits, and taking into account (17.5), (31) becomes

$$W_t^* = (1 + r - c)\frac{(1 + r - c)^t - 1}{r - c} \qquad (32)$$

When *t=1* lead us to case 5 in section three. At time one, the wealth of the worker will be: *1+r-c*. At this point it is convenient to note that the saving in period one is higher in this case than when the fee is over flow. Nevertheless, the base of the exponent *t* is lower than in (30), so it seems plausible thinking that in a given moment in time it could be more convenient to the worker switching from one Afore that charges over real return toward one with charge over flow. Let us see case three.

Case 3: fee over Balance

Now $\delta = \partial = 1,$ so (28') becomes

$$W_t^{**} = \varepsilon(1 + r)\frac{(1 + r)^t \varepsilon^t - 1}{(1 + r)\varepsilon - 1} \qquad (33)$$

Assuming again zero profits, considering (17.3), and after some algebra, we get:

$$W_t^{**} = (1 + r - c)\frac{(1 + r - c)^t - 1}{r - c} \qquad (34)$$

This is equal to (32). From here is obvious that, **it is equivalent charging over real return or over balance,** this is because in both cases the interest rate is implied in the fee.

We have seen that in the early stages it is better for the worker to be in an Afore that charges over balance or real return. But in a certain moment it will be convenient to change to another with fee over flow. So the strategy for the rational worker must be: at the beginning of the labor life choose the Afore that charges over real return or balance, when the wealth in period *t* is equal to the another option ($W_t^{**} = W_t$), change to another Afore that charges over flow. Now let us see what could be the strategy for a worker that already has (or had) a wealth when the new system started working.

5.2 Finding the optimal wealth path for any worker at time *t*

As we have seen before, the wealth of the worker at time *t* will be given by his wealth at time *t-1*, plus the saving in *t* plus the interest minus the commissions the afore charges. In other words:

$$W_t = (W_{t-1} + 1 - f)(1 + r(1-\rho))(1-s) \tag{35}$$

where:

W_t: Wealth at period *t*.

Let us note that (35) is equivalent to (24).

This is the general equation of the saving process. As we can see in table one, in real life the Afores were charging, at most, only for two of those concepts, actually there was one Afore that used to charge only over return. So let us analyze the possible cases:

Case 1) Charging over flow and balance:

$$W_t = (W_{t-1} + 1 - f)(1 + r)(1-s) \tag{35.1}$$

Case 2) Charging over flow and real return:

$$W_t = (W_{t-1} + 1 - f)(1 + r(1-\rho)) \tag{35.2}$$

Case 3) Charging over real return and balance:

$$W_t = (W_{t-1} + 1)(1 + r(1 - \rho))(1 - s) \qquad (35.3)$$

Case 4) Charging over flow:

$$W_t = (W_{t-1} + 1 - f)(1 + r) \quad (35.4)$$

Case 5) Charging over real return:

$$W_t = (W_{t-1} + 1)(1 + r(1 - \rho)) \qquad (35.5)$$

Case 6) Charging over balance:

$$W_t = (W_{t-1} + 1)(1 + r)(1 - s) \qquad (35.6)$$

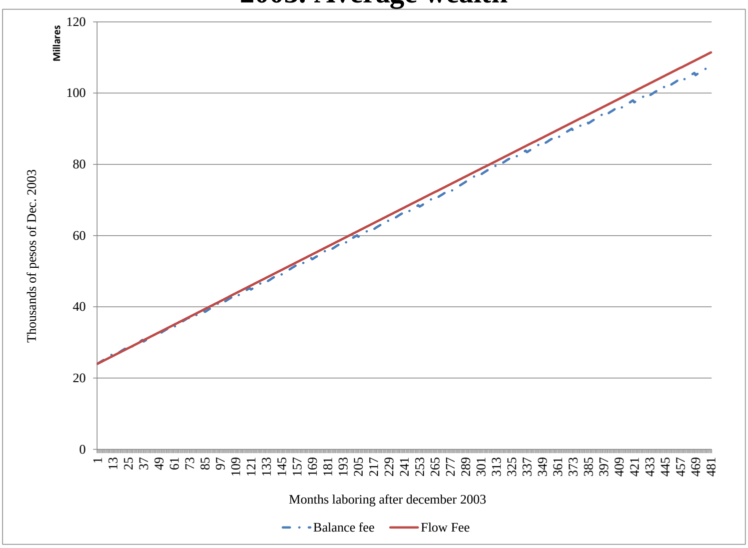
Before March 2008, the Afores were charging only over flow and balance, some of them only over flow, from that month, they are charging only over balance. In any case, what a rational worker should do is: applying the previous formulas and verify which Afore gives him the highest wealth, keep in that Afore up to the moment in what the wealth obtained from another one is higher.

As an example we took the average worker of December 2003. He was 34 years old, his Afore wealth was $24,028.00; wage $2,834.2; the Social Contribution (SC) was $43.64 all above in terms of pesos of July of 1997 and for the wage and SC, we are referring in monthly base. Although there are two accounts, one for housing and another for retirement, is this last one we have dealt with in this paper, so we focus just in it. The Social Contribution is the saving paid directly by federal government and is equal to a 5% of a minimum wage of July 1997; this SC is not considered when charging over flow, but it is when charging over return or balance.

In this exercise we are taking three cases: a) the average worker and the average wealth in his Afore Account; b) average worker and zero saving, so in a way we are assuming that this person is going to save for the first time in his life; finally, c) average worker, but three times the average wealth, in this case we want to show what is happening with those workers that already have a huge amount in their saving accounts.

We are assuming that the wage does not change for all the period we are doing this example. We took the average balance fee and the average flow fee of December 2003, reported by the Consar in its web page. They take the equivalent balance fee and the equivalent flow fee, I am taking both cases to show up what happens with the worker's wealth path.

Chart 2. Wealth path for the average worker of December, 2003. Average wealth



Source: made with data from Banco de México and Consar.

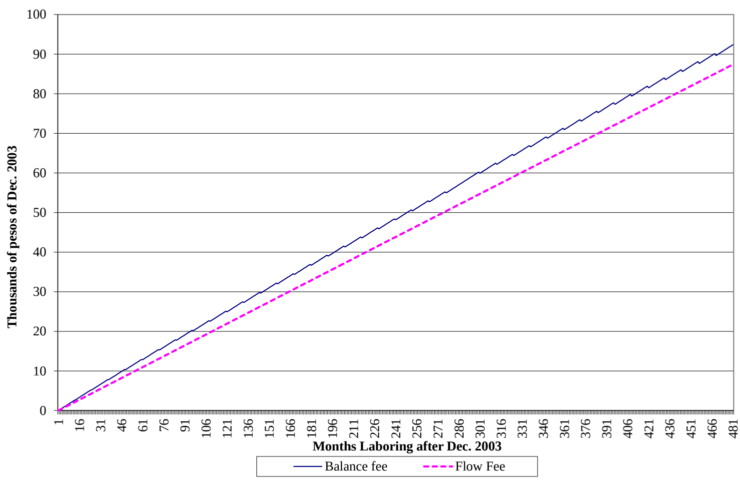
We can see that in the early stages of the saving process there is not really a big difference in either way of charging the fee, it is just in the long term the fee over flow allows the wealth to grow at faster rates. The reason is simple: the higher the wealth, the higher the fee when it is going to be charged over balance.

The opposite happens when the worker has zero balance: if he is starting saving, for him is better to be charged over balance (since it is close to zero) and not over flow. We can see in chart 3 that his wealth grows up faster when the fee is over balance rather than over flow.

Finally, let us see what happens when the worker already has a huge wealth in his account. In this example, I am still assuming the average worker, but with three times the average wealth.

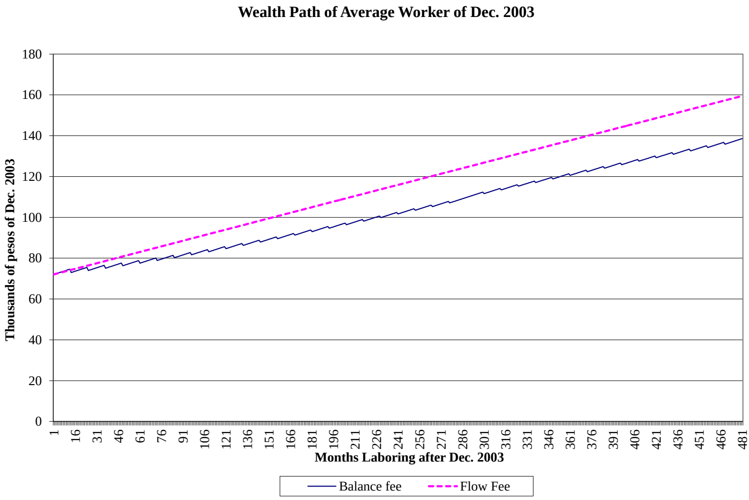
We can see that in this case the best option for the worker is a fee over flow. Again, it is not difficult figure out why: when the wealth is large, the fee over balance will be large too, so in this case it will be better for the worker to be charged over flow.

Chart 3. Wealth path for the average worker of December, 2003. Zero wealth



Source: made with data from Banco de México and Consar.

Chart 4. Wealth path for the average worker of December, 2003. Three times de average wealth



Source: made with data from Banco de México and Consar.

The above graphs show us that charging over just one concept could not be the best for the workers. Having two options, that allow him to choose an Afore that charges over flow or over balance could be better for the worker in the long term, provided that they have full information.

5.3 Lessons of the dynamic approach

Assuming that the expected return is equal for all Afores, it is clear that the simplest way of charging is by flow. Nevertheless, this deal is good for the worker only if his wealth in period *t-1* is big enough to generate a huge charge over balance or over return. When the wealth is in the early stages of saving, is better for the worker case 5 or 6, meaning when the fee is over real return or balance.

Given the actual market structure, it is possible to find an optimal path for the wealth of the worker, it will depend on the actual wealth and in the fees the Afores are charging. When the reform took place, there where several workers in different stages of their cycle life, so there was a huge diversity of wealth. In this sense, it was good for them having different options to choose. Since in every period of time there are diversity of workers, with different cumulative wealth, different wage and different age, then, in order to find an optimal path, should be several Afores that charge over flow or over balance.

The need of more information

So far it seems pretty clear that when the worker has information regarding his account, it requires simple arithmetic to find out the optimal path for his wealth. Information could lead toward more efficient market structure. That must be one of the most important jobs of the entity in charge of regulating the mutual funds industry (CONSAR).

If the worker has enough information, he will follow an optimal path, but for that he needs market structure with Afores charging for at least two kind of fees, among them over flow. But as long as they do not have enough information or even do not know the wealth belongs to them, the market structure could produce undesirable outcomes, meaning that in twenty five years or more, the workers could not have enough money to pay their own pension. We could be in a situation with rich Afores but poor pensioners. The government has a minimum pension guarantee, so under the worst case scenario, it could be an important fiscal cost to be paid for the tax payers. This risk could be reduced by given more information to the consumer and, therefore, having a more competitive market structure.

5.4 The Afores behavior

So far we have analyzed what should be the aftermath of having a Bertrand competition. I have stated the need for more information to *really* have this market structure. However, it seems that in spite of the lack of information on the side of the workers, the Afores are moving toward less expensive fees, so we could see a reduction in price. As it is shown in the next table.

Table 1. Fees over balance, last month of year

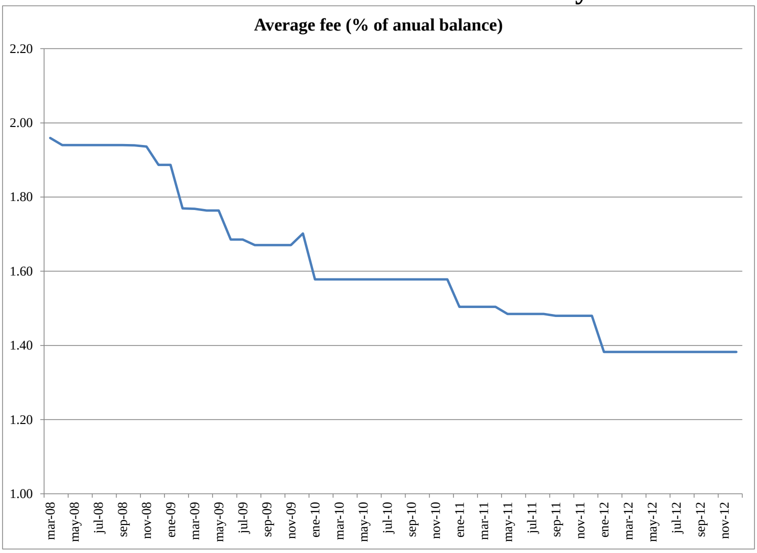
Fee on balance (annual %)	march 08	dec 08	dec 09	dec 10	dec 11	dec 12
Afirme Bajío	1.70	1.70	1.70	1.51	1.51	1.50
Ahorra Ahora	3.00	3.00	N/A	N/A	N/A	N/A
Argos	1.18	1.17	N/A	N/A	N/A	N/A
Azteca	1.96	1.96	1.96	1.96	1.67	1.52
Banamex	1.84	1.84	1.75	1.58	1.45	1.28
Bancomer	1.47	1.47	1.47	1.45	1.40	1.28
Banorte Generali	1.71	1.71	1.71	1.58	1.48	N/A
Coppel	3.30	3.30	1.94	1.81	1.70	1.59
HSBC	1.77	1.77	1.77	1.61	N/A	N/A
Inbursa	1.18	1.18	1.18	1.18	1.17	1.17
Invercap	2.48	2.48	1.93	1.73	1.72	1.59
Ixe	1.83	1.83	N/A	N/A	N/A	N/A
Metlife	2.26	2.26	1.89	1.74	1.69	1.54
PensionISSSTE	N/A	1.00	1.00	1.00	1.00	0.99
Principal	2.11	2.05	1.94	1.79	1.52	1.48
Profuturo GNP	1.96	1.96	1.92	1.70	1.53	1.39
Scotia	2.33	1.98	1.88	N/A	N/A	N/A
SURA	1.74	1.74	1.74	1.61	1.48	1.31
XXI Banorte	1.45	1.45	1.45	1.42	1.40	1.33
Average	1.96	1.89	1.70	1.58	1.48	1.38

Source: CONSAR http://www.consar.gob.mx/SeriesTiempo/Series.aspx?cd=7&cdAlt=False [Consultado el 18 de enero, 2013].
Notes:
N / A.- Not available.

Source: Consar’s web page, http://www.consar.gob.mx/SeriesTiempo/Series.aspx?cd=7&cdAlt=False, [Consultado el 18 de enero de 2013].

Table 1 shows that some Afores have reduced the fee. It seems plausible to think that when workers have more information, the fees will reduce even more. As a matter of fact, my point of view is that eventually they will reach the Bertrand Equilibrium.

Chart 5. Fee over balance in a monthly base



Source: http://www.consar.gob.mx/SeriesTiempo/Series.aspx?cd=7&cdAlt=False, [Consultado el 18 de enero de 2013].

A final issue we will discuss is the market concentration. The typical announcement of the Consar is that new firms have provoked a reduction in the fees. This view is the one of Cournot competence. We have computed the Herfindahl index from December of 1998 to 2006 to measure the competition of the industry. Let us remember that the higher the value of the index, the higher the concentration of the industry on a few firms.

First column of table 2 shows the number of Afores existing in that month. Second is the computation of the H index assuming that each afore has the same number of workers. H real refers to the actual value of H, meaning measuring market concentration. Finally last column shows the number of firms that would exist in the market given de real value of H assuming that the accounts are perfectly distributed between the Afores.

Table 2. Herfindhal Index of Afore industry

Month	N	H perfect distribution	H real	N Equivalent
dic-97	17	588.2	1101.8	9.1
dic-98	14	714.3	1114.9	9.0
dic-99	13	769.2	1108.2	9.0
dic-00	13	769.2	1100.7	9.1
dic-02	13	769.2	925.7	10.8
dic-03	11	909.1	1096.2	9.1
dic-04	12	833.3	1086.3	9.2
dic-05	13	769.2	1029.0	9.7
dic-06	16	625.0	959.0	10.4
dic-07	21	476.2	881.8	11.3
dic-08	21	476.2	851.5	11.7
dic-09	19	526.3	918.8	10.9
dic-10	16	625.0	926.5	10.8
dic-11	15	666.7	909.2	11.0
dic-12	14	714.3	954.3	10.5

Source: http://www.consar.gob.mx/SeriesTiempo/Enlace.aspx?md=5&nl=2 [consulted January 18, 2013].

The last two columns show that in spite of having more firms in the industry, the concentration is higher. Actually, even when there are several firms, the market structure has a equivalent of having less. This means that it is not really very important to have several firms and the workers distributed in many of them. But the aim of this paper is even beyond that: even if there are a few firms, with full information on the worker's side, the market would reach the Bertrand equilibrium, meaning, a perfect competitive market solution. The trajectory of fees and the concentration of market shows that.

Conclusions

In a world with complete information, it is enough to allow the existence of two firms to reach competitive solution in a pension fund management market. It implies that it is not necessary to have plenty of firms to reach perfect competition equilibrium. Maybe this is why the entity in charge of regulating the Afores allows them to compete just in prices. The Chilean case shows that after twenty years, a few firms only survive; the Mexican market has shown that several Afores had merging with others, reducing the final number of them. Given the Chilean experience, it is plausible to think that in the future the market will have just a few firms, but that does not necessarily imply that they will have market power.

The model developed in this paper helps to understand that the best thing for the workers is to incentive competition in prices and that two firms will be enough to reach perfect competition equilibrium. The differences in the fees can have a different aftermath in the worker's wealth: if the firms charge over flow in the early stages of saving process, the worker receives less net return while charging over balance lets them to get higher revenue. The reason is simple: one is charged at the beginning of the period and the other at the end, so the worker is better off when he is charged at the end (meaning over balance) rather than at the beginning (over flow). But in a given moment the things will reverse. So the optimal wealth path is that that allows him to have the maximum wealth along all the period of time.

Having uncertainty increases the fee and the worker pays for it. But the basic results still hold. The dynamic approach allows us to see that the fee over balance is equivalent to the fee over real return, and allow confirming that this kind of fee is better at the beginning of the saving process, but not at the late stages. When the wealth grows, and the fee over balance is higher than over flow, it is a moment to change to another Afore that charges over flow.

In a dynamic approach we could found that it is equivalent to charge over balance or over return. In those terms, to simplify the system it could be convenient to regulate to allow only two kind of fees: over flow and over balance or return.

Another conclusion is that more Afores does not necessarily implies more competence. The model developed here shows that the industry really does not need plenty of firms as long as they compete in prices and the workers have more information.

Finally, it could be possible to be in an optimal wealth path if and only if the worker has full information. Right now plenty of workers do not even know which Afore is managing their wealth. Perhaps this is the most important issue in the development of this market, since given the asymmetric information, the Afores are taking advantage of this fact, charging fees that allows them to have positive profits. It is true that the knowledge is power so the way of giving the workers more wealth at the retirement time, is teaching them now that they have wealth and can choose between several Afores and, therefore, several fees.

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