Consistency in valuation:
A practical guide*

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Consistencia en la valoración: una guía práctica

ABSTRACT

Practitioners and teachers easily break some consistency rules when conducting or teaching valuation of assets, which may lead to different results by different methods. In this short note we present a practical guide to call attention to the most frequently broken consistency rules. Firstly, they have to do with the consistency in matching of the cash flows. Secondly, with the proper expression for the cost of levered equity, and different formulations for the weighted average cost of capital, for finite cash flows. Thirdly, with the consistency between the terminal value and growth. In this article we deal with the first two, and leave any considerations about terminal value for a subsequent note, considering for now the terminal value as given. We show that, keeping this consistency, all methods lead to the same value. We illustrate this by a simple example. In the Appendices we show some algebraic derivations

Key words: cash flows, valuation, levered value, levered equity value, weighted average cost of capital.

RESUMEN

Practicantes y docentes rompen muy fácilmente algunas reglas de consistencia cuando aplican o enseñan el tema de la valoración de activos, lo cual puede llevar a resultados diferentes al utilizar diferentes métodos. En esta nota presentamos una guía práctica para llamar la atención sobre las reglas de consistencia que se infringen con mayor frecuencia. Ellas tienen que ver, en primer lugar, con la consistencia al relacionar los flujos de caja. En segundo lugar, con la expresión adecuada para el costo del capital propio cuando se tiene apalancamiento y las diferentes formulaciones para el costo promedio ponderado para flujos finitos. En tercer lugar, con la consistencia entre el valor terminal y el crecimiento. En este artículo tocamos las dos primeras y dejamos para una nota posterior cualquier consideración acerca del valor terminal, que por ahora lo asumiremos como dado. Mostramos que, al cuidar estas consistencias, todos los métodos llevan al mismo resultado, lo cual ilustramos mediante un ejemplo simple. En los apéndices se muestran algunas derivaciones algebraicas.

Palabras clave: flujos de caja, valoración, valor con apalancamiento, valor del capital propio con apalancamiento, costo promedio ponderado de capital.

*We wish to thank joseph tham from duke university for the fruitful discussions we had on this paper. any error or mistake is our entire responsibility.

1. INTRODUCCION

Practitioners and teachers easily break some consistency rules when conducting or teaching valuation of assets. In this short and simple note we present a practical guide to call attention to the most frequently broken consistency rules. Firstly, they have to do with the consistency in the matching of the cash flows, this is, the free cash flow (FCF), the cash flow to debt (CFD), the cash flow to equity (CFE), the capital cash flow (CCF) and the tax savings or tax shield (TS). Secondly, they have to do with the proper expression for the cost of levered equity, K_e and different formulations for the weighted average cost of capital, WACC, for finite cash flows. Thirdly, although not covered in this article, they have to do with the consistency between the terminal value and growth for the FCF and the terminal value and growth for the CFE. .

Consequently, emphasis will be placed in showing that convergence exists among the most known and used valuation methods, which means that they all lead to the same value, as long as the proper formulas for Ke and WACC are used in each case. This is an important aim, considering that frequently in practice quite different values are shown, attributing the difference to the method that was used and, in some cases, to rounding errors.

According to this main objective, the paper will not discuss the fundamentals or the validity of those methods, whose mechanics have been published in many textbooks and manuals. This would be the subject of another type of publication. In this process we do not pretend either to describe the many factors that may influence the value of required parameters like K_u (unlevered cost of equity), K_d (cost of debt), or K_e (cost of levered equity). We assume that the firm can somehow determine how much, in average, the resources obtained by debtors cost, and can also define the proper discount rate for cash flows if the firm had no debt. As far as the cost of equity with debt, K_e we develop a general formula and adapt it to each particular situation. Economists can discuss and support many variables that might influence interest rates and consequently the cost of debt. In a similar manner Capital Market theorists can argue about the facts that determine what a fair rate of return for the stocks of a given firm is, in particular considering the risk involved. We depart from a known and widely mentioned formula (3a) derived in Appendix A and that depends on the unlevered cost of equity, the cost of debt and the proper discount rate for the tax shield, both assumed as given after considering all economic factors that may affect them. Again, we do not pretend to pose a discussion about its validity or alternatives to calculate Ke like the CAPM, which has been the object of several critics and modifications. We only show that, with its appropriate use, all methods stated for valuation lead to the same value, contrary to what other authors and practitioners have erroneously sustained. We do not pretend that stating these parameters is unimportant, on the contrary, valuation of a firm must include a hard effort to assume an appropriate value for these starting parameters, taking into consideration the present conditions of the economy and the firm, as well as the assumptions about the future conditions.

In the example we test for equality of results of the main methods mentioned in the literature and which are used in practice for valuation by the most important investment banks or consulting firms.

As to the terminal value, we depart from a simplified model in favor of the brevity of the paper and to stick to the main objective expressed above. Many aspects like inflation and perpetual growth enter into the determination of this value, and a big controversy can be found as to the right way to include these variables. Considering this point would be enough matter for a separate extensive article or series of articles. However, in Appendix B we derive the formulation for Terminal value as a growing perpetuity.

This note is organized as follows: In Section 2, we present the consistency of cash flows and values according to the Modigliani and Miller (1958, 1963) propositions. Based on these ideas we define consistency in terms of cash flows and values. In Section 3 we show the different expressions for Ke, traditional WACC and general WACC for finite cash flows. In Section 4 we show the example that illustrates the consistency, showing also how to deal with the circularity in calculations by starting with an arbitrary value of WACC, in this case zero. In Section 5 we conclude. In the Appendix A we derive the basic algebraic expressions.

2. THE MODIGLIANI–MILLER (M&M) PROPOSALS

The basic idea is that the value of a firm does not depend on how the stakeholders finance it (see Modigliani & Miller, 1958, 1963). This is the stockholders (equity) and creditors (liabilities to banks, bondholders, etc.) They proposed that with perfect market conditions, (perfect and complete information, no taxes, etc.) the capital structure does not affect the value of the firm because the equity holder can borrow and lend and thus determine the optimal amount of leverage. The capital structure of the firm is the combination of debt and equity in it. In other words, the following relation exists under perfect conditions:

$$VUL = VL = VEquity + VDebt$$

- (1a)

Where VUL and VL are the unlevered and levered Values of the firm, and VEquity and VDebt are the value of the equity and the value of the debt. In terms of cash, this means also that.

FCF = CFD + CFE

- (1b)

Where FCF is the free cash flow, CFD is the cash flow to debt, and CFE the cash flow to equity. TS), and equation (1a) turns into

VL = VUL + VTS = VDebt + VEquity

- (2a)

The corresponding cash flow relationship is

FCF + TS = CFD + CFE

- (2b)

Or

FCF = CFD + CFE – TS

- (2c)

Where TS is the tax shield for the period or the subsidy the firm receives for paying interest.

The sum of what the owners of the capital obtain is named as Capital Cash Flow (CCF) and is equal to the sum of the CFD and the CFE.

How do we discount these cash flows to obtain values? In Table 1 we indicate which discount rate to use for each cash flow.

Table 1

Correspondence between cash flows and discount rates.

Cash flow	Discount rate	To calculate
CFD	Cost of debt, Kd	Market value of debt
CFE	Cost of levered equity, Ke	Market value of equity
FCF	WACCFCF	Levered market value of firm
FCF	Cost of unlevered equity, Ku	Unlevered market value of firm
TS	The appropriate discount rate for TS, ψ	The market value of the TS
CCF	WACCCCF	Levered market value of firm

Our purpose is to provide the correct procedures and expressions for the different inputs in valuing a cash flow and to guarantee the consistency between the cash flows and the market values according to what we presented above.

3. THE PROPER COST OF CAPITAL: WHICH DISCOUNT RATE FOR TS WE CAN USE

In this section we list the proper definitions for Ke and WACC for finite cash flows taking into account which discount rate we use for TS, ψ. We will consider only two values for ψ = Ku and Kd. Taggart (1989) has developed some expressions with this purposes. Some of our results coincide with Taggart's. He considers corporate and personal taxes; we only consider corporate taxes.

On the other hand, Inselbag and Kaufold (1997) have tackled the problem of choosing between discounted cash flow, DCF methods and the Adjusted Present Value, APV method. This is a false choice because as we show, all methods give identical answers. They show that the two approaches give the same value. However, Inselbag and Kaufold say, "one must already have calculated the firm's value (using APV or some other means) to be able to derive the discount rates necessary to value the firm using the WACC method". They conclude that the APV is better than the DCF when the debt schedule is given. This is misleading in two senses: one, they mix methods because they disregard the possibility of solving the circularity posed by the relationship between value and discount rates and second, as a consequence, they say that "one must already have calculated the firm's value" in order to know the WACC. Also, they derive the value of the firm assuming target leverage and assume the Miles and Ezzell (1980) approach for calculating the value of the TS. However, when using the M&E approach they assume (as M&E do) that the value of the firm is known and hence the value of debt is known. This is not true. In any case the value of the firm is not known in advance, but they assume that.

In this case they conclude that the DCF is better than the APV because the APV generates circularity and has to be solved using iterations. The circularity problem is very easy to solve (see Tham & Velez, 2004; Vélez & Tham, 2009). In a spreadsheet construct the circularity relation and go to Tools →Options →Activate Iterations (for Excel 2003); for Excel 2007 go to the Office Button →Excel Options →Formulas →Enable (tick) Iterations. It must be said that the iterative process is something that when using a spreadsheet the user does not even notice.

In the following paragraphs we list the different cases for Ke, for the WACC for the FCF and for the CCF. Each of these sets of formulas is presented to be applied to the CFE, to the FCF and to the CCF.

The general expression for Ke is

$$Ke_i = Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{EL_{i-1}} - (Ku_i - \psi_i)\frac{VTS_{i-1}}{EL_{i-1}}$$

- (3a)

Where Ke is the levered cost of equity, Ku is the unlevered cost of equity, Kd is the cost of debt, D is the market value of debt, E is the market value of equity, ψ is the discount rate for the TS and VTS is the present value of the TS at ψ. (See Appendix A for derivation.)

From this expression we can derive the formulation when ψi is Kd or Ku:

When ψi is Kui

If ψ_i is Ku_i the third term in the right hand side (RHS) of equation 3a vanishes, and the expression for Ke is

$$Ke_i = Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{EL_{i-1}}$$

- (3b)

When ψi is Kui

If ψ_i is Kd_i then

$$Ke_i = Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{EL_{i-1}} - (Ku_i - \psi_i)\frac{VTS_{i-1}}{EL_{i-1}}$$

- (3c)

or

$$Ke_i = Ku_i + (Ku_i - Kd_i) \quad \begin{matrix} D_{i-1} - VTS_{i-1} \\ EL_{i-1} \;\; EL_{i-1} \end{matrix}$$

- (3d)

When we examine the weighted average cost of capital, WACC, we can handle the problem in a similar way. Let us call the WACC to be used for FCF WACCFCF, and let us start with the traditional formula:

$$Kd_i(1 - T) \quad \frac{D_{i-1}}{EL_{i-1}} + \frac{Ke_iE_{i-1}}{VL_{i-1}}$$

When using this expression for WACC we have to be careful and use the proper formulation for Ke depending on the assumption about ψ.

We have to warn the reader about the correctness of the traditional WACC. The previous expression shows the typical and best known formulation for WACC, but it has to be said that this formulation is valid only for a precise and special case: when there is enough earnings before interest and taxes (EBIT) to fully earn the TS, when the only source of TS is the interest charges and that taxes are paid the same year as accrued. To cover deviations from this special case we can use the following more general formulation for WACC:

$$WACC_iGeneral = Ku_i - \quad \frac{TS_i}{VL_{i-1}} \; -(Ku_i - \psi_i) \; \frac{VTS_{i-1}}{VL_{i-1}}$$

- (3e)

(See Appendix A for derivation).

If $\psi_i = Ku_i$ the third term in the RHS of equation 3e vanishes, and we obtain

$$WACC_iGeneral = Ku_i - \quad \frac{TS_i}{VL_{i-1}}$$

- (4)

If $\psi_i = Kd_i$, we obtain

$$WACC_i General = Ku_i - \frac{TS_i}{VL_{i-1}} - Ku_i - Kd_i) \frac{VTS_{i-1}}{VL_{i-1}}$$

- (5)

When the traditional WACC and the general WACC can be used? It depends on what happens to the tax savings. There are situations when the tax savings cannot be earned in full a given year due to a very low Earnings Before Interest and Taxes, EBIT, (and there exist legal provision for losses carried forward, LCF) or the tax savings are not earned in the current year because taxes are not paid the same year as accrued or there are other sources different from interest charges that generate tax savings, such as adjustments for inflation to the financial statements (Tham & Vélez, 2004; Vélez & Tham, 2003). When these anomalies occur the traditional formulation for the traditional WACC cannot be used.

On the other hand let us apply the same analysis for the appropriate general WACC for CCF

The general formula for the WACCCCF is as follows.

$$WACC_i General = Ku_i - (Ku_i - \psi_i) \frac{VTS_{i-1}}{VL_{i-1}}$$

- (6)

(See Appendix A, equations A17 to A20 for derivation)

When replacing the corresponding ψ_I, the following expressions are obtained:

For $\psi i = Kui$ $\quad$ $WACC_i General = Ku_i$

- (6a)

For $\psi i = Kui$ $\quad$ $WACC_i General = Ku_i - (Ku_i - Kd_i) \frac{VTS_{i-1}}{VL_{i-1}}$

- (6b)

4. EXAMPLE FOR MATCHING METHODS

In a similar manner as Taggart (1977) showed that, under a number of simplifying assumptions, three methods of capital budgeting procedures were equivalent, we present five firm valuation methods that match in calculating values, when all consistency rules are maintained. These methods are the traditional WACC, the “General” WACC for discounting the FCF, CFE with Ke, the CCF and the APV.

In this Section we show an example to illustrate the ideas presented in the paper. The formulae are the ones developed in the paper; however they are made explicit in the exhibits corresponding to each calculation.

In this example we consider some outstanding debt at year N. See Table 2. Assume we have the following information:

1) The cost of debt, Kd is constant and equal to 10%. Constant rates for cost of debt are a simplification; in reality that cost is not constant for several reasons. One of them is that the firm might have several sources of financial debt with different rates. The correct way to consider the cost of debt is to divide the interest charges for the period by the initial debt of the period. The implicit assumption here is that the contractual cost of debt is identical to the market cost of debt.
2) The risk free rate, Rf, is 8%.
3) The market risk premium MRP is 5%.
4) The unlevered beta βu is 1.4.
5) The tax rate is 40% and taxes are paid the same year as accrued.
6) There is enough EBIT to earn the TS.
7) To set a terminal value at year five, it is assumed that after year five a Free Cash Flow of 13,76 will maintain growing at perpetuity at 2.91% annually, that inflation will be 8% per year, and that WACC will keep a value of 12.1% from then on, that is [(1.121/1.08)-1] = 3.8% net of inflation. Under those conditions the terminal value will be 13.76*[(1+0.0291)/0.038] = 373. The justification of this formula is found in Appendix B, equation B6.
8) In addition we know that the debt balance and the CFE is as follows.

Table 2

Debt balance and CFE.

Year	2009	2010	2011	2012	2013	2014
Debt balance	23.00	31.00	38.00	46.00	46.00	46.00
CFE		14.00	16.00	17.00	10.00	11.00

The CFE can be derived from the cash budget looking at the dividends and/or repurchase of equity and/or new equity investment.

From this information we can make some estimates, as follows:

1) We can estimate the unlevered cost of equity, Ku, as follows, using the CAPM: $Ku = Rf + \beta u \times MRP = 8\% + 1.4 \times 5\% = 15.0\%$

2) The interest payments and the TS can be calculated.The interest charge is $I = Kd \times D_{t-1}$ and the TS is $T \times I$. In fact the interest charges can be read directly from the debt schedule or the cash budget. See Table 3.

Table 3

Interest charges and tax savings.

Year	2009	2010	2011	2012	2013	2014
Interest charges		2.3	3.1	3.8	4.6	4.6
TS		0.92	1.24	1.52	1.84	1.84

3) The CFD can be calculated from the debt balance and the interest charges.The principal payment PPMT, is the difference between two successive debt balances, $PPMT_t = D_{t-1} - Dt$. In fact, the PPMT can be read directly from the debt schedule or the cash budget. See Table 4.

Table 4

Debt balance, principal payment, interest and CFD.

Year	2009	2010	2011	2012	2013	2014
Debt balance	23.00	31.00	38.00	46.00	46.00	46.00
PPMT		-8.00	-7.00	-8.00	0.00	-94.50
Interest charges I		2.30	3.10	3.80	4.60	4.60
CFD = PPMT + I		-5.70	-3.90	-4.20	4.60	4.60

4.1. Finite cash flows and Kd as the discount rate for the TS

Now we will derive the firm and equity values using several methods and assuming finite cash flows and Kd as the discount rate for the TS.

Using the WACCGeneral, from equation (5) to discount FCF we calculate solving the circularity, the levered value of the firm and the equity. The present value of the TS is calculated using Kd as the discount rate.

For this example we initialize the procedure calculating values with WACC equal to 0%. After having those calculated values we proceed to introduce the formula for the WACC. We apply this procedure in those cases where needed. In Table 5a we show the temporary results for WACC equal to 0%.

Table 5a

Discount rate for the TS is Kd; levered values calculated with FCF and general WACCFCF(temporary).

2009	2009	2010	2011	2012	2013	2014
FCF		7.38	10.86	11.28	12.76	13.76
Terminal value						373.00
TS		0.92	1.24	1.52	1.84	1.84
Value of TS	5.40	5.02	4.28	3.19	1.67	
$WACC_{General}$						
Total levered value	429.0400	421.6600	410.8000	399.5200	386.7600	373.00
Levered equity	406.0400	390.6600	372.8000	353.5200	340.7600	

Note: we show the final results with the circularity solved. See Table 5b.

Table 5b

Discount rate for the TS is Kd; levered values calculated with FCF and general WACCFCF (final).

2009	2009	2010	2011	2012	2013	2014
FCF		7.38	10.86	11.28	12.76	13.76
Terminal value						373.00
TS		0.92	1.24	1.52	1.84	1.84
Value of TS	5.40	5.02	4.28	3.19	1.67	
$WACC_{General}$		14.48%	14.41%	14.38%	14.35%	14.43%
Total levered value	227.0319	252.5166	278.0430	306.7352	337.9858	373.00
Levered equity	204.0319	221.5166	240.0430	260.7352	291.9858	

Now using the CCF and the WACCCCF we calculate the same values. From equation

6b) we know that the general $WACC_i$General is $Ku_i - (Ku_i - Kd_i)\dfrac{VTS_{i-1}}{VL_{i-1}}$

In Table 6a we show the temporary values when WACCCCF is 0%.

Table 6a

Discount rate for the TS is Kd; levered values calculated with CCF
and WACC for CCF (temporary).

2009	2009	2010	2011	2012	2013	2014
CCF = FCF + TS = CFD + CFE		8.30	12.10	12.80	14.60	15.60
Terminal value						373.00
TS		0.92	1.24	1.52	1.84	1.84
Value of TS	5.40	5.02	4.28	3.19	1.67	-
$WACC_iCCF = Ku_i - (Ku_i - Kd_i)\frac{VTS_{i-1}}{VL_{i-1}}$						
Total levered value	436.4000	428.1000	416.0000	403.2000	388.6000	373.00
Levered equity	413.4000	397.1000	378.0000	357.2000	342.6000	

In Table 6b we show the final values.

As it should be, the values for the firm and equity match.

As the conditions required for using the traditional WACC formulation and the FCF are fulfilled, we present the values calculated using it. From equation (3d) we know that the correct formulation for Ke with finite cash flows and Kd as the discount rate for TS is

$$Ku_i + (Ku_i - Kd_i)\frac{D_{i-1} - VTS_{i-1}}{EL_{i-1}\ \ EL_{i-1}}$$

and we defined traditional WACC as

$$Kd_i(1 - T)\frac{D_{i-1}}{EL_{i-1}} + \frac{Ke_iE_{i-1}}{VL_{i-1}}$$

Table 6b

Discount rate for the TS is Kd; levered values calculated with CCF
and WACC for CCF (final).

2009	2009	2010	2011	2012	2013	2014
CCF = FCF + TS = CFD + CFE		8.30	12.10	12.80	14.60	15.60
Terminal value						373.00
TS		0.92	1.24	1.52	1.84	1.84
Value of TS	5.40	5.02	4.28	3.19	1.67	-
$WACC_i^{CCF} = Ku_i - (Ku_i - Kd_i)\frac{VTS_{i-1}}{VL_{i-1}}$		14.88%	14.90%	14.92%	14.95%	
Total levered value	227.0319	252.5166	278.0430	306.7352	337.9858	373.00
Levered equity	204.0319	221.5166	240.0430	260.7352	291.9858	

In Table 7a we show the temporary calculations assuming WACC equal to zero.

Table 7a

Discount rate for TS is Kd; levered values calculated with FCF
and traditional WACCFCF (temporary).

2009	2009	2010	2011	2012	2013	2014
CCF = FCF + TS = CFD + CFE		7.38	10.86	11.28	12.76	13.76
Terminal value						373.00
Kd(1−T)		6.00%	6.00%	6.00%	6.00%	6.00%
Kd(1−T)D%		0.32%	0.44%	0.56%	0.69%	0.71%
TS		0.92	1.24	1.52	1.84	1.84
Value of TS	5.40	5.02	4.28	3.19	1.67	-
$Ke = Ku_i + (Ku_i - Kd_i)\left(\frac{DL_{i-1}}{EL_{i-1}} - \frac{VTS_{i-1}}{EL_{i-1}}\right)$		15.22%	15.33%	15.45%	15.61%	15.65%
KeE%		14.40%	14.21%	14.02%	13.81%	13.79%
Traditional WACC						
Total levered value	429.0400	421.6600	410.8000	399.5200	386.7600	373.00
Levered equity	406.0400	390.6600	372.8000	353.5200	340.7600	

In Table 7b we show the final calculations after solving the circularity.

Again, as expected, the calculated values match.

Now we calculate the TV for the CFE as the TV for the FCF minus the outstanding debt and the equity value (and total levered value) using the CFE.

In Table 8a we show the temporary solution when Ke is zero.

After solving the circularity, we find the final value. This is shown in Table 8b.

Again, the values match.

Now we examine the Adjusted Present value approach to check if keeping the assumptions the values match. We have to realize that the APV when Kd is the discount rate for TS is identical to the present value of the CCF.

Table 7b

Discount rate for TS is Kd; levered values calculated with FCF and traditional WACCFCF (final).

2009	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**
FCF		7.38	10.86	11.28	12.76	13.76
Terminal value						373.00
Kd(1−T)		6.00%	6.00%	6.00%	6.00%	6.00%
Kd(1−T)D%		0.61%	0.73%	0.82%	0.90%	0.82%
TS		0.92	1.24	1.52	1.84	1.84
Value of TS	5.40	5.02	4.28	3.19	1.67	-
$Ke = Ku_i + (Ku_i - Kd_i)\frac{DL_{i-1}}{EL_{i-1}} - \frac{VTS_{i-1}}{EL_{i-1}}$		15.43%	15.59%	15.70%	15.82%	15.76%
KeE%		13.87%	13.67%	13.56%	13.45%	13.79%
Traditional WACC		14.48%	14.41%	14.38%	14.35%	14.43%
Total levered value	227.0319	252.5166	278.0430	306.7352	337.9858	373.00
Levered equity	204.0319	221.5166	240.0430	260.7352	291.9858	

Table 8a

Discount rate for TS is Kd; calculation of levered values using TVCFE as TVFCF minus debt (temporary).

2009	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**
FCF		14.00	16.00	17.00	10.00	11.00

$TV_{CFE} = TV_{FCF}$ - debt						327.00
TS		0.92	1.24	1.52	1.84	1.84
Value of TS	5.40	5.02	4.28	3.19	1.67	-
$Ke = Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{EL_{i-1}} - \frac{VTS_{i-1}}{EL_{i-1}}$						
Total levered value	395.0000	381.0000	365.0000	348.0000	338.0000	327.00
Levered equity	418.0000	412.0000	394.0000	260.7352	384.0000	

Table 8b

Discount rate for TS is Kd; calculation of levered values using TVCFE as TVFCF minus debt (final).

2009	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**
FCF		14.00	16.00	17.00	10.00	11.00
$TV_{CFE} = TV_{FCF}$ - debt						327.00
TS		0.92	1.24	1.52	1.84	1.84
Value of TS	5.40	5.02	4.28	3.19	1.67	-
$Ke = Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{EL_{i-1}} - \frac{VTS_{i-1}}{EL_{i-1}}$		15.43%	15.59%	15.70%	15.82%	15.76%
Total levered value	204.0319	221.5166	240.0430	260.7352	291.9858	327.00
Levered equity	227.0319	252.5166	278.0430	306.7352	337.9858	

Once again the values match because we have used consistent formulations for every method.

At this moment we are not surprised that the values match because in all the methods we have kept the same assumptions and we have used the correct formulations for each set of assumptions. As we can see, all the calculated values, including the calculations for the cost of capital match when we use the consistent assumptions and the proper formulations for each case. This is shown in Table 9.

Table 9

Discount rate Kd; levered values calculated with APV.

Year	2009	2010	2011	2012	2013	2014
FCF		7.38	10.86	11.28	12.76	13.76
Terminal value						373.00
TS			1.24	1.52	1.84	1.84
PV(FF at Ku)	36.1826	34.2300	28.5045	21.5002	11.9652	
PV(at Kd)	5.4024	5.0226	4.2849	3.1934	1.6727	
VP(V at Ku)	185.4469	213.2640	245.2536	282.0416	324.3478	373.00
Total APV	227.0319	252.5166	278.0430	306.7352	337.9858	
Equity = Total APV − Debt	204.0319	221.5166	240.0430	260.7352	291.9858	

4.2. Finite cash flows and Ku as the discount rate for the TS

Now we will derive the firm and equity values using several methods and assuming finite cash flows and Ku as the discount rate for the TS.

In Table 10a we show the preliminary results before solving the circularity.

Table 10a

Discount rate for the TS is Ku; levered values calculated with FCF and general WACC (temporary).

Year	2009	2010	2011	2012	2013	2014
FCF		7.38	10.86	11.28	12.76	13.76
Terminal value						373.00
TS		0.92	1.24	1.52	1.84	1.84
WACCGeneral $= Ku_i - \frac{TS_i}{VL_{i-1}}$						
Total levered value	429.0400	421.6600	410.8000	399.5200	386.7600	373.00
Levered equity	406.0400	390.6600	372.8000	353.5200	340.7600	

Using the WACCGeneral from equation (4), WACC General, $Ku_i - \frac{TS_i}{VL_{i-1}}$ we calculate solving the circularity, the levered value of the firm and the equity. Using the data from the example we calculate the levered values using the FCF and the general WACC. This is shown in Table 10b.

Now using the CCF and the WACCCCF we calculate the same values. From equation (6a) we know that the WACCCCF is K_u. In this case there is no circularity.

As we expected, the levered values match. See Tables 10b and 11. It is not strange because we have used the same assumptions and the correct formulations in each case.

Table 10b

Discount rate for the TS is Ku; levered values calculated with FCF and general WACC (final).

Year	2009	2010	2011	2012	2013	2014
FCF		7.38	10.86	11.28	12.76	13.76
Terminal value						373.00
TS		0.92	1.24	1.52	1.84	1.84
WACCGeneral $= Ku_i - \frac{TS_i}{VL_{i-1}}$		14.59%	14.51%	14.45%	14.40%	14.46%
Total levered value	226.3334	251.9834	277.6809	306.5331	337.9130	373.00
Levered equity	203.3334	220.9834	239.6809	260.5331	291.9130	

Table 11

Discount rate for TS is Ku; levered values calculated with CCF and WACC for CCF.

Year	2009	2010	2011	2012	2013	2014
FCF		7.38	10.86	11.28	12.76	13.76
Terminal value TV						373.00
TS		0.92	1.24	1.52	1.84	1.84
CC F = FCF + TS		8.30	12.10	12.80	14.60	15.60
WACCCCF=Ku		15.00%	15.00%	15.00%	15.00%	15.00%
Levered value	226.3334	251.9834	277.6809	306.5331	337.9130	373.0000
L evered equity	203.3334	220.9834	239.6809	260.5331	291.9130	

In Table 12a we show the preliminary results before circularity is solved. In this case we assume WACC equal to zero.

Table 12a

Discount rate for the TS is Ku; levered values calculated with FCF and traditional WACC (temporary).

Year	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**
FCF with TV_{FCF}		7.38	10.86	11.28	12.76	13.76
$Kd \times (1-T)$		6.00%	6.00%	6.00%	6.00%	6.00%
$Kd \times D\% \times (1-T)$		0.32%	0.44%	0.56%	0.69%	0.71%
$Ke = Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{EL_{i-1}}$		15.28%	15.40%	15.51%	15.65%	15.67%
KeE%		14.46%	14.26%	14.07%	13.85%	13.81%
Traditional WACC						
Levered value	429.0400	421.6600	410.8000	399.5200	386.7600	373.00
Levered equity	406.0400	390.6600	372.8000	353.5200	340.7600	

Again, as the conditions required the use of the traditional WACC formulation and the FCF, we present the values calculated using it. From Equation (3b) we know what the correct formulation for Ke is according to the discount rate for the TS. Table 12b shows the final results after circularity is solved.

Table 12b

Discount rate for the TS is Ku; levered values calculated with FCF and traditional WACC (final)

Year	**2009**	**2010**	**2011**	**2012**	**2013**	**2014**
FCF with TV_{FCF}		7.38	10.86	11.28	12.76	13.76
$Kd \times (1-T)$		6.00%	6.00%	6.00%	6.00%	6.00%
$Kd \times D\% \times (1-T)$		0.61%	0.74%	0.82%	0.90%	0.82%
$Ke = Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E_{i-1}}$		15.57%	15.70%	15.79%	15.88%	15.79%
Ke%		13.98%	13.77%	13.63%	13.50%	13.64%
Traditional WACC		14.59%	14.51%	14.45%	14.40%	14.46%
Levered value	226.3334	251.9834	277.6809	306.5331	337.9130	

Levered equity	203.3334	220.9834	239.6809	260.5331	291.9130	373.00

As expected, the levered values coincide. Now we calculate the TV for the CFE as the TV for the FCF minus the outstanding debt and the equity value (and total levered value) using the CFE.

Table 13a shows the preliminary values before solving the circularity. We assume Ke equal to zero.

Table 13a

Discount rate for the TS is Ku; levered values with the CFE (temporary).

Year	2009	2010	2011	2012	2013	2014
$TV_{CFE} = TV_{FCF}$ - debt		14.00	16.00	17.00	10.00	11.00
$Ke = Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E_{i-1}}$						327.00
Levered equity	395.0000	381.0000	365.0000	348.0000	338.0000	327.00
Debt	23.0000	31.0000	38.0000	46.0000	46.0000	46.0000
Total levered value	418.0000	412.0000	403.0000	394.0000	384.0000	373.0000

Table 13b shows the final results after solving circularity.

Again, the values match.

Now we examine the Adjusted Present value approach to check if keeping the assumptions the values match. We have to realize that the APV when Ku is the discount rate for TS is identical to the present value of the CCF, because the CCF = FCF + TS = CFD + CFE. Then when calculating the present value of the CCF as the right hand side of the equation, it is the same as calculating the present value of the left hand side.

Table 13b

Discount rate for the TS is Ku; levered values with the CFE.

Year	2009	2010	2011	2012	2013	2014

CFE		14.00	16.00	17.00	10.00	11.00
$TV_{CFE} = TV_{FCF} - debt$						327.00
$Ke = Ku_i + (Ku_i - Kd_i)\frac{D_{i-1}}{E_{i-1}}$		15.57%	15.70%	15.79%	15.88%	15.79%
Levered equity	203.3334	220.9834	239.6809	260.5331	291.9130	327.00
Debt	23.00	31.00	38.00	46.00	46.00	46.00
Total levered value	226.3334	251.9834	277.6809	306.5331	337.9130	373.00

Once again the values match because we have used consistent formulations for each method. In Table 14 we show the final test: all previous results match the APV with discount rate of TS equal to Ku.

Table 14

Discount rate for TS is Ku; levered values calculated with APV.

Year	2009	2010	2011	2012	2013	2014
FCF						
Terminal value						373.00
TS						
PV(FCF at Ku)	36.1826	34.2300	28.5045	21.5002	11.9652	
PV(TS at Ku)	4.7039	4.4895	3.9229	2.9913	1.6000	
VP(VT a Ku)	185.4469	213.2640	245.2536	282.0416	324.3478	373.00
Total APV	226.3334	251.9834	277.6809	306.5331	337.9130	
Equity = Total APV − Debt	203.3334	220.9834	239.6809	260.5331	291.9130	

5. CONCLUDING REMARKS

We have shown that there exist expressions for the WACC and Ke under different assumptions regarding the discount rate for the TS and we have applied them to an example, in such a way that we have shown that when done properly, we can arrive to the same correct levered values using the FCF, the CCF or the CFE.

From the exploration of the calculated levered values we observe that the values obtained when we assume Kd as the discount rate are higher than those calculated with Ku as the discount rate (227.03 as compared with 226.33). This is not a surprise because the later assumption does not have the (1 – T) factor in the calculation of the Ke and this makes the discount rates higher than when we use the (1 – T) factor in the calculation of Ke. Another explanation can be seen examining the APV when the discount rate for TS is Kd and Ku.

When Ku is used to discount TS its present value is lower than when discounted with Kd. We have to remember that Ku is larger than Kd.

The values with the two different assumptions for the discount rate for the TS are quite close. In pointing out these differences we are not claiming that one assumption is correct and the other is incorrect. That is a debate that has not concluded.

We have thus presented a summary of the proper relationships for cash flows and the appropriate cost of capital and shown that all methods lead to the same result, only depending on the discount rate assumed for TS.

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Recepción del artículo: 15/09/2008
Envío evaluación a autores: 15/12/2008
Recepción correcciones: 18/02/2009
Aceptación artículo: 30/06/2009

Appendix A

Valuation Using WACC.

This appendix is based on Tham and Velez–Pareja (2002).

General expression for the return to levered equity Ke_i

We briefly derive the general algebraic expressions for the cost of capital that is applied to finite cash flows. Firstly, we show that in general the return to levered equity Ke_i is a function of ψ_i, and this is a most important point. Secondly, we derive the general expressions for the WACCs.

First we use a general expression for the present value, PV, of any cash flow, CF, (a basic tenet of finance) as follows,

$$PV_{i-1} = \frac{CF_i + PV_i}{1 + K_i}$$

- (6b)

Where PV is the present value at period i and i − 1, CF_i is the cash flow at i and K_i is the discount rate at i. This is one of what is called the basic tenets of finance.

As can be seen in this equation (A1a), it is a practical and simple way to calculate the present value from one period back to the previous one, even when discount rates are not constant and hence traditional formulas or even the Excel or any other spreadsheet time value of money functions will not work.

Solving for CF_i we have

$$CF_i = PV_{i-1} \times (1+K_i) - PV_i$$

- (A1b)

Second, using (A1b), we write the main equations as follows.

$$FCF_i = VUn_{i-1} \times (1 + Ku_i) - VUn_i$$

- (A1c)

We express the FCF (unlevered cash flow) as a function of the unlevered value that is obtained when we discount the FCF with the cost of unlevered equity, Ku.

$CFE_i = EL_{i-1} \times (1 + Ke_i) - EL_i$

- (A2)

We express the CFE (cash flow to equity) as a function of the equity levered value that is obtained when we discount the CFE with the cost of levered equity, Ke.

$CFD_i = D_{i-1} \times (1 + Kd_i) - D_i$

- (A3)

We express the CFD (cash flow to debt) as a function of the debt value that is obtained when we discount the CFD with the cost of debt, Kd.

$TS_i = VTS_{i-1} \times (1 + \psi_i) - VTS_i$

- (A4)

We express the TS (tax savings) as a function of the TS value that is obtained when we discount the TS with the assumed discount rate for TS, ψ. We know that,

$FCF_i + TS_i = CFE_i + CFD_i$

- (A5a)

hence,

$VUn_i + VTS_i = EL_i + D_i$

- (A5b)

and

$VUn_i = EL_i + D_i - VTS_i$

- (A5c)

To obtain the general expression for the Ke, substitute equations A1 to A4 into equation A5a.

$VUn_{i-1} \times (1 + Ku_i) - VUn_i + VTS_{i-1} \times (1 + \psi_i) - VTS_i$

$= EL_{i-1} \times (1 + Ke_i) - EL_i + D_{i-1} \times (1 + Kd_i) - D_i$

- (A6)

We simplify applying (A5b) and obtain,

$VUn_{i-1} \times Ku_i + VTS_{i-1} \times \psi_i = EL_{i-1} \times Ke_i + D_{i-1} \times Kd_i$

- (A7)

Solving for the return to levered equity and using A5c.

$EL_{i-1} \times Ke_i = (EL_{i-1} + D_{i-1} - VTS_{i-1}) \times Ku_i + VTS_{i-1} \times \psi_i - D_i\text{-}1 \times Kd_i$

- (A8)

Collecting terms and rearranging, we obtain,

$EL_{i-1} \times Ke_i = EL_{i-1} \times Ku_i + (Ku_i - Kd_i) \times D_{i-1} - (Ku_i - \psi_i) \times VTS_{i-1}$

- (A9)

Solving for the return to levered equity, we obtain,

$$Ke_i = Ku_i + (Ku_i - Kdi)\frac{D_{i-1}}{EL_{i-1}} - (Ku_i - \psi\ \frac{VTS_{i-1}}{EL_{i-1}}$$

- (A10)

General WACC applied to the FCF

We can express the FCF as follows

$$FCF_i = Ku \times VUn_{i-1} = WACC_i \times VL_{i-1}$$

- (A11)

Let $WACCGeneral_i$ be the general WACC that is applied to the FCF in year i.

We call WACCGeneral just to differentiate this formulation from the traditional WACCFCF.We would prefer to express the WACCFCF as the more general expression as we have presented here.

From (A11) and from (2c in the body of the paper) we have

$$VL_{i-1} \times WACCGeneral_i = D_{i-1} \times Kd_i - TS_i + EL_{i-1} \times Ke_i$$

- (A12)

As we know,

$$FCF_i + TS_i = CFD_i + CFE_i$$

- (A13a)

and

$FCF = VUn_{i-1} \times Ku_i$

- (A13b)

and

$TS = VTS_{i-1} \times \psi_I$

- (A13c)

we combine (A12), (A13b) and (A13c) to obtain

$VL_{i-1} \times WACCGeneral_i = VUn_{i-1} \times Ku_i + VTS_{i-1} \times \psi_i - TS_i$

- (A13d)

We know that

$VL_{i-1} = VUn_{i-1} + VTS_{i-1}$

- (A13e)

In words, the difference between the levered value and the unlevered value is the value of the tax savings. Hence

$VUn_{i-1} = VL_{i-1} - VTS_{i-1}$

- (A13f)

Combining (A13f) and (A13d) we have

$VL_{i-1} \times WACCGeneral_i = (VL_{i-1} - VTS_{i-1)} \times Ku_i + VTS_{i-1} \times \psi_i - TS_i$

- (A14)

$VL_{i-1} \times WACCGeneral_i = VL_{i-1} \times Ku_i - (Ku_i - \psi_i) \times VTS_{i-1} - TS_i$

- (A15)

Solving for the WACC in equation A15, we obtain,

$WACC_iGeneral= Kui - (Kui - \psi) \quad \frac{D_{i-1}}{EL_{i-1}} \quad -(Ku_i- \psi \quad \frac{VTS_{i-1}}{EL_{i-1}} - \frac{TS_i}{VL_{i-1}}$

- (A16)

General WACC applied to the CCF

We know that the CCF is equal to the sum of the FCF and the TS.

$CCF_i = FCF_i + TS_i = CFD_i + CFE_i$

- (A17a)

Using (A1c), (A2), (A3) and (A4) we have from (A17a) and letting $WACCGeneralCCF_i$ be the general WACC applied to the CCF.

$VL_{i-1} \times (1+WACCGeneralCCF_i) - VL_i = VUn_{i-1} \times (1 + Ku_i) - VUn_i + VTS_{i-1}$

- (A17b)
  $\times (1 + \psi_i) - VTS_i = D_{i-1} \times (1 + Kd_i) - D_i + EL_{i-1} \times (1 + Ke_i) - EL_i$

Simplifying, organizing and using the equation for values (A5b) we have

$VL_{i-1} \times(1+ WACCGeneralCCF_i) = VUn_{i-1} \times (1 + Ku_i) + VTS_{i-1} \times (1 + \psi_i) =$
$D_{i-1} \times (1 + Kd_i) + EL_{i-1} \times (1 + Ke_i)$

- (A17c)

Simplifying again we drop VL_{i-1}, VUn_{i-1}, VTS_{i-1}, D_{i-1} and EL_{i-1} and we obtain

$$VL_{i-1} \times WACCGeneralCCF_i = VUn_{i-1} \times Ku_i + VTS_{i-1} \times \psi_i$$

- (A18)

$$VL_{i-1} \times WACCGeneralCCF_i = VL_{i-1} \times Ku_i - (Ku_i - \psi_i) \times VTS_{i-1}$$

- (A19)

Solving for the WACCGeneralCCF, we obtain,

$$WACC_i GeneralCFC = Kui - (Kui - \psi_i) \frac{VTS_{i-1}}{VL_{i-1}}$$

- (A19)

Appendix B

Derivation of formula for Terminal Value.

If we expect FCF will grow we need to invest some portion of the FCF. Otherwise, we will say that FCF will grow from the thin air. We assume that the FCF has included an amount of investment that keeps the level of investment necessary to grant that the FCF is constant. If this is true, and we wish FCF grows at a given rate, we have to invest an amount that grants that growth.

The amount of extra investment will be a function of the desired growth. We assume that the fraction of FCF invested to keep the real growth is

$$\text{Fraction of FCF invested} = \frac{g}{wacc}$$

- (B1)

Where g is the real growth rate and wacc is the deflated Weighted Average Cost of Capital.

$$TV = \frac{FCFN \times (1+G) \times \left(1 - \frac{g}{wacc}\right)}{WACC - G}$$

- (B2)

$$TV = \frac{FCFN \times (1+G) \times \frac{wacc-g}{wacc}}{WACC - G}$$

- (B3)

Where G is the nominal growth rate and WACC is the nominal Weighted Average Cost of Capital. But

$$WACC - G = (wacc-g) \times (1+Infl)$$

- (B4)

And

$$(1+g) = (1+G)/(1+ Infl)$$

- (B5)

Where infl is inflation rate. Hence,

$$TV = \frac{FCF_N(1+g)}{WACC}$$

- (B6)